CHAPTER 8 Circle

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. If A and B are points in the plane such that $PA/PB = k$ (constant) for all P on a given circle, then the value of k cannot be equal to .... *(1982 - 2 Marks)*
2. The points of intersection of the line $4x - 3y - 10 = 0$ and the circle $x^2 + y^2 - 2x + 4y - 20 = 0$ are .... and .... *(1983 - 2 Marks)*
3. The lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to the same circle. The radius of this circle is .... *(1984 - 2 Marks)*
4. Let $x^2 + y^2 - 4x - 2y - 11 = 0$ be a circle. A pair of tangents from the point (4, 5) with a pair of radii form a quadrilateral of area .... *(1985 - 2 Marks)*
5. From the origin chords are drawn to the circle $(x - 1)^2 + y^2 = 1$. The equation of the locus of the mid-points of these chords is .... *(1985 - 2 Marks)*
6. The equation of the line passing through the points of intersection of the circles $3x^2 + 3y^2 - 2x + 12y - 9 = 0$ and $x^2 + y^2 + 6x + 2y - 15 = 0$ is .... *(1986 - 2 Marks)*
7. From the point $A(0, 3)$ on the circle $x^2 + 4x + (y - 3)^2 = 0$, a chord AB is drawn and extended to a point M such that $AM = 2AB$. The equation of the locus of M is .... *(1986 - 2 Marks)*
8. The area of the triangle formed by the tangents from the point (4, 3) to the the circle $x^2 + y^2 = 9$ and the line joining their points of contact is .... *(1987 - 2 Marks)*
9. If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that common chord is of maximum length and has a slope equal to 3/4, then the coordinates of the centre of C_2 are .... *(1988 - 2 Marks)*
10. The area of the triangle formed by the positive x-axis and the normal and the tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is, ... *(1989 - 2 Marks)*
11. If a circle passes through the points of intersection of the coordinate axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$, then the value of λ = ... *(1991 - 2 Marks)*
12. The equation of the locus of the mid-points of the circle $4x^2 + 4y^2 - 12x + 4y + 1 = 0$ that subtend an angle of $2\pi/3$ at its centre is .... *(1993 - 2 Marks)*
13. The intercept on the line y = x by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle with AB as a diameter is .... *(1996 - 1 Mark)*
14. For each natural number k, let C_k denote the circle with radius k centimetres and centre at the origin. On the circle C_k, α-particle moves k centimetres in the counter-clockwise direction. After completing its motion on C_k, the particle moves to C_{k+1} in the radial direction. The motion of the particle continues in this manner. The particle starts at (1, 0). If the particle crosses the positive direction of the x-axis for the first time on the circle C_n then n = ... *(1997 - 2 Marks)*
15. The chords of contact of the pair of tangents drawn from each point on the line $2x+y=4$ to circle $x^2+y^2 = 1$ pass through the point .... *(1997 - 2 Marks)*

B True / False

1. No tangent can be drawn from the point (5/2, 1) to the circumcircle of the triangle with vertices $(1, \sqrt{3})$ $(1, -\sqrt{3})$, $(3, -\sqrt{3})$. *(1985 - 1 Mark)*
2. The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 - 6x + 2y = 0$. *(1989 - 1 Mark)*

C MCQs with One Correct Answer

1. A square is inscribed in the circle $x^2 + y^2 - 2x + 4y + 3 = 0$. Its sides are parallel to the coordinate axes. The one vertex of the square is *(1980)*
   (a) $(1 + \sqrt{2}, -2)$ (b) $(1 - \sqrt{2}, -2)$
   (c) $(1, -2 + \sqrt{2})$ (d) none of these
2. Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their points of intersection and the point (1, 1) is *(1980)*
   (a) $x^2 + y^2 - 6x + 4 = 0$ (b) $x^2 + y^2 - 3x + 1 = 0$
   (c) $x^2 + y^2 - 4y + 2 = 0$ (d) none of these

3. The centre of the circle passing through the point (0, 1) and touching the curve $y = x^2$ at (2, 4) is *(1983 - 1 Mark)*

(a) $\left(\frac{-16}{5}, \frac{27}{10}\right)$ (b) $\left(\frac{-16}{7}, \frac{53}{10}\right)$

(c) $\left(\frac{-16}{5}, \frac{53}{10}\right)$ (d) none of these

4. The equation of the circle passing through (1, 1) and the points of intersection of $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ is *(1983 - 1 Mark)*

(a) $4x^2 + 4y^2 - 30x - 10y - 25 = 0$
(b) $4x^2 + 4y^2 + 30x - 13y - 25 = 0$
(c) $4x^2 + 4y^2 - 17x - 10y + 25 = 0$
(d) none of these

5. The locus of the mid-point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is *(1984 - 2 Marks)*

(a) $x + y = 2$ (b) $x^2 + y^2 = 1$
(c) $x^2 + y^2 = 2$ (d) $x + y = 1$

6. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its centre is *(1988 - 2 Marks)*

(a) $2ax + 2by - (a^2 + b^2 + k^2) = 0$
(b) $2ax + 2by - (a^2 - b^2 + k^2) = 0$
(c) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$
(d) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - k^2) = 0$.

7. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then *(1989 - 2 Marks)*

(a) $2 < r < 8$ (b) $r < 2$ (c) $r = 2$ (d) $r > 2$

8. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle of area 154 sq. units. Then the equation of this circle is *(1989 - 2 Marks)*

(a) $x^2 + y^2 + 2x - 2y = 62$
(b) $x^2 + y^2 + 2x - 2y = 47$
(c) $x^2 + y^2 - 2x + 2y = 47$
(d) $x^2 + y^2 - 2x + 2y = 62$

9. The centre of a circle passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$ is *(1992 - 2 Marks)*

(a) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{3}{2}\right)$ (c) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, -2^{\frac{1}{2}}\right)$

10. The locus of the centre of a circle, which touches externally the circle $x^2 + y^2 - 6x - 6y + 14 = 0$ and also touches the y-axis, is given by the equation: *(1993 - 1 Marks)*

(a) $x^2 - 6x - 10y + 14 = 0$ (b) $x^2 - 10x - 6y + 14 = 0$
(c) $y^2 - 6x - 10y + 14 = 0$ (d) $y^2 - 10x - 6y + 14 = 0$

11. The circles $x^2 + y^2 - 10x + 16 = 0$ and $x^2 + y^2 = r^2$ intersect each other in two distinct points if *(1994)*

(a) $r < 2$ (b) $r > 8$ (c) $2 < r < 8$ (d) $2 \le r \le 8$

12. The angle between a pair of tangents drawn from a point P to the circle $x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$ is 2α. The equation of the locus of the point P is *(1996 - 1 Mark)*

(a) $x^2 + y^2 + 4x - 6y + 4 = 0$ (b) $x^2 + y^2 + 4x - 6y - 9 = 0$
(c) $x^2 + y^2 + 4x - 6y - 4 = 0$ (d) $x^2 + y^2 + 4x - 6y + 9 = 0$

13. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the x–axis, then *(1999 - 2 Marks)*

(a) $p^2 = q^2$ (b) $p^2 = 8q^2$ (c) $P^2 < 8q^2$ (d) $p^2 > 8q^2$.

14. The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have co-ordinates (3,4) and (–4, 3) respectively, then $\angle QPR$ is equal to *(2000S)*

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

15. If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$, $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is *(2000S)*

(a) 2 or $-\frac{3}{2}$ (b) -2 or $-\frac{3}{2}$ (c) 2 or $\frac{3}{2}$ (d) -2 or $\frac{3}{2}$

16. Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then the locus of the centroid of the triangle PAB as P moves on the circle is *(2001S)*

(a) a parabola (b) a circle
(c) an ellipse (d) a pair of straight lines

17. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then $2r$ equals *(2001S)*

(a) $\sqrt{PQ.RS}$ (b) $(PQ + RS)/2$
(c) $2PQ.RS/(PQ + RS)$ (d) $\sqrt{(PQ^2 + RS^2)}/2$

18. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets a straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is *(2002S)*

(a) 4 (b) $2\sqrt{5}$ (c) 5 (d) $3\sqrt{5}$

19. The centre of circle inscribed in square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$, is *(2003S)*

(a) (4, 7) (b) (7, 4) (c) (9, 4) (d) (4, 9)

20. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is *(2004S)*

(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 3 (d) 2

21. A circle is given by $x^2 + (y-1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is *(2005S)*

(a) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \le 0\}$
(b) $\{(x, y) : x^2 + (y - 1)^2 = 4\} \cup \{(x, y) : y \le 0\}$
(c) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \le 0\}$
(d) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

22. Tangents drawn from the point $P(1, 8)$ to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is ***(2009)***
(a) $x^2 + y^2 + 4x - 6y + 19 = 0$
(b) $x^2 + y^2 - 4x - 10y + 19 = 0$
(c) $x^2 + y^2 - 2x + 6y - 29 = 0$
(d) $x^2 + y^2 - 6x - 4y + 19 = 0$

23. The circle passing through the point $(-1, 0)$ and touching the y-axis at $(0, 2)$ also passes through the point. ***(2011)***
(a) $\left(-\frac{3}{2}, 0\right)$ (b) $\left(-\frac{5}{2}, 2\right)$ (c) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (d) $(-4, 0)$

24. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is ***(2012)***
(a) $20(x^2 + y^2) - 36x + 45y = 0$
(b) $20(x^2 + y^2) + 36x - 45y = 0$
(c) $36(x^2 + y^2) - 20x + 45y = 0$
(d) $36(x^2 + y^2) + 20x - 45y = 0$

D MCQs with One or More than One Correct

1. The equations of the tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$, are ***(1988 - 2 Marks)***
(a) $x = 0$ (b) $y = 0$
(c) $(h^2 - r^2)x - 2rhy = 0$ (d) $(h^2 - r^2)x + 2rhy = 0$

2. The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is ***(1998 - 2 Marks)***
(a) 0 (b) 1 (c) 3 (d) 4

3. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$, $S(x_4, y_4)$, then ***(1998 - 2 Marks)***
(a) $x_1 + x_2 + x_3 + x_4 = 0$ (b) $y_1 + y_2 + y_3 + y_4 = 0$
(c) $x_1x_2x_3x_4 = c^4$ (d) $y_1y_2y_3y_4 = c^4$

4. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are) ***(JEE Adv. 2013)***
(a) $x^2 + y^2 - 6x + 8y + 9 = 0$ (b) $x^2 + y^2 - 6x + 7y + 9 = 0$
(c) $x^2 + y^2 - 6x - 8y + 9 = 0$ (d) $x^2 + y^2 - 6x - 7y + 9 = 0$

5. A circle S passes through the point $(0, 1)$ and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then ***(JEE Adv. 2014)***
(a) radius of S is 8 (b) radius of S is 7
(c) centre of S is $(-7, 1)$ (d) centre of S is $(-8, 1)$

6. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point $(1, 0)$. Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s) ***(JEE Adv. 2016)***
(a) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (b) $\left(\frac{1}{4}, \frac{1}{2}\right)$
(c) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (d) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

E Subjective Problems

1. Find the equation of the circle whose radius is 5 and which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ at the point $(5, 5)$. ***(1978)***

2. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose that the tangents at the points $B(1, 7)$ and $D(4. -2)$ on the circle meet at the point C. Find the area of the quadrilateral $ABCD$. ***(1981 - 4 Marks)***

3. Find the equations of the circle passing through $(-4, 3)$ and touching the lines $x + y = 2$ and $x - y = 2$. ***(1982 - 3 Marks)***

4. Through a fixed point (h, k) secants are drawn to the circle $x^2 + y^2 = r^2$. Show that the locus of the mid-points of the secants intercepted by the circle is $x^2 + y^2 = hx + ky$. ***(1983 - 5 Marks)***

5. The abscissa of the two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $x^2 + 2px - q^2 = 0$. Find the equation and the radius of the circle with AB as diameter. ***(1984 - 4 Marks)***

6. Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 of diameter 6. If the centre of C_1 lies in the first quadrant, find the equation of the circle C_2 which is concentric with C_1 and cuts intercepts of length 8 on these lines. ***(1986 - 5 Marks)***

7. Let a given line L_1 intersects the x and y axes at P and Q, respectively. Let another line L_2, perpendicular to L_1, cut the x and y axes at R and S, respectively. Show that the locus of the point of intersection of the lines PS and QR is a circle passing through the origin. ***(1987 - 3 Marks)***

8. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$ is inscribed in a triangle which has two of its sides along the co-ordinate axes. The locus of the circumcentre of the triangle is $x + y - xy + k(x^2 + y^2)^{1/2} = 0$. Find k. ***(1987 - 4 Marks)***

9. If $\left(m_i, \frac{1}{m_i}\right)$, $m_i > 0$, $i = 1, 2, 3, 4$ are four distinct points on a circle, then show that $m_1m_2m_3m_4 = 1$ ***(1989 - 2 Marks)***

10. A circle touches the line $y = x$ at a point P such that $OP = 4\sqrt{2}$, where O is the origin. The circle contains the point $(-10, 2)$ in its interior and the length of its chord on the line $x + y = 0$ is $6\sqrt{2}$. Determine the equation of the circle. ***(1990 - 5 Marks)***

11. Two circles, each of radius 5 units, touch each other at $(1, 2)$. If the equation of their common tangent is $4x + 3y = 10$, find the equation of the circles. ***(1991 - 4 Marks)***

12. Let a circle be given by $2x(x - a) + y(2y - b) = 0$, $(a \neq 0, b \neq 0)$. Find the condition on a and b if two chords, each bisected by the x- axis, can be drawn to the circle from $\left(a, \frac{b}{2}\right)$. ***(1992 - 6 Marks)***

13. Consider a family of circles passing through two fixed points A (3,7) and B (6, 5). Show that the chords in which the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinate of this point. *(1993 - 5 Marks)*

14. Find the coordinates of the point at which the circles $x^2 + y^2 - 4x - 2y = -4$ and $x^2 + y^2 - 12x - 8y = -36$ touch each other. Also find equations common tangents touching the circles in the distinct points. *(1993 - 5 Marks)*

15. Find the intervals of values of a for which the line $y + x = 0$ bisects two chords drawn from a point $\left(\frac{1+\sqrt{2}a}{2}, \frac{1-\sqrt{2}a}{2}\right)$ to the circle $2x^2 + 2y^2 - (1+\sqrt{2}a)x - (1-\sqrt{2}a)y = 0$. *(1996 - 5 Marks)*

16. A circle passes through three points A, B and C with the line segment AC as its diameter. A line passing through A intersects the chord BC at a point D inside the circle. If angles DAB and CAB are α and β respectively and the distance between the point A and the mid point of the line segment DC is d, prove that the area of the circle is

$$\frac{\pi d^2 \cos^2 \alpha}{\cos^2 \alpha + \cos^2 \beta + 2\cos\alpha \cos\beta \cos(\beta - \alpha)}$$

(1996 - 5 Marks)

17. Let C be any circle with centre $(0, \sqrt{2})$. Prove that at the most two rational points can be there on C. (A rational point is a point both of whose coordinates are rational numbers.) *(1997 - 5 Marks)*

18. C_1 and C_2 are two concentric circles, the radius of C_2 being twice that of C_1. From a point P on C_2, tangents PA and PB are drawn to C_1. Prove that the centroid of the triangle PAB lies on C_1. *(1998 - 8 Marks)*

19. Let T_1, T_2 be two tangents drawn from (–2, 0) onto the circle $C : x^2 + y^2 = 1$. Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles, when taken two at a time. *(1999 - 10 Marks)*

20. Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin O to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA. *(2001 - 5 Marks)*

21. Let C_1 and C_2 be two circles with C_2 lying inside C_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of the centre of C. *(2001 - 5 Marks)*

22. For the circle $x^2 + y^2 = r^2$, find the value of r for which the area enclosed by the tangents drawn from the point P (6, 8) to the circle and the chord of contact is maximum. *(2003 - 2 Marks)*

23. Find the equation of circle touching the line $2x + 3y + 1 = 0$ at (1, – 1) and cutting orthogonally the circle having line segment joining (0, 3) and (–2, –1) as diameter. *(2004 - 4 Marks)*

24. Circles with radii 3, 4 and 5 touch each other externally. If P is the point of intersection of tangents to these circles at their points of contact, find the distance of P from the points of contact. *(2005 - 2 Marks)*

G Comprehension Based Questions

PASSAGE-1

$ABCD$ is a square of side length 2 units. C_1 is the circle touching all the sides of the square $ABCD$ and C_2 is the circumcircle of square $ABCD$. L is a fixed line in the same plane and R is a fixed point.

1. If P is any point of C_1 and Q is another point on C_2, then $\frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$ is equal to *(2006 - 5M, –2)*

(a) 0.75 (b) 1.25 (c) 1 (d) 0.5

2. If a circle is such that it touches the line L and the circle C_1 externally, such that both the circles are on the same side of the line, then the locus of centre of the circle is *(2006 - 5M, –2)*

(a) ellipse (b) hyperbola
(c) parabola (d) pair of straight line

3. A line L' through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts L' at T_2 and T_3 and AC at T_1, then area of $\Delta T_1T_2T_3$ is *(2006 - 5M, –2)*

(a) $\frac{1}{2}$ sq. units (b) $\frac{2}{3}$ sq. units
(c) 1 sq. units (d) 2 sq. units

PASSAGE-2

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

4. The equation of circle C is *(2008)*

(a) $(x - 2\sqrt{3})^2 + (y-1)^2 = 1$

(b) $(x - 2\sqrt{3})^2 + \left(y + \frac{1}{2}\right)^2 = 1$

(c) $(x - \sqrt{3})^2 + (y+1)^2 = 1$

(d) $(x - \sqrt{3})^2 + (y-1)^2 = 1$

5. Points E and F are given by *(2008)*

(a) $\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right),(\sqrt{3},0)$ (b) $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right),(\sqrt{3},0)$

(c) $\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right),\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$ (d) $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right),\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

6. Equations of the sides QR, RP are *(2008)*

(a) $y=\frac{2}{\sqrt{3}}x+1,\ y=-\frac{2}{\sqrt{3}}x-1$

(b) $y=\frac{1}{\sqrt{3}}x,\ y=0$

(c) $y=\frac{\sqrt{3}}{2}x+1,\ y=-\frac{\sqrt{3}}{2}x-1$

(d) $y=\sqrt{3}x,\ y=0$

PASSAGE-3

A tangent PT is drawn to the circle $x^2+y^2=4$ at the point $P(\sqrt{3},1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2+y^2=1$. *(2012)*

7. A possible equation of L is

(a) $x-\sqrt{3}y=1$ (b) $x+\sqrt{3}y=1$

(c) $x-\sqrt{3}y=-1$ (d) $x+\sqrt{3}y=5$

8. A common tangent of the two circles is

(a) $x=4$ (b) $y=2$

(c) $x+\sqrt{3}y=4$ (d) $x+2\sqrt{2}y=6$

H Assertion & Reason Type Questions

1. Tangents are drawn from the point (17, 7) to the circle $x^2+y^2=169$.

STATEMENT-1 : The tangents are mutually perpendicular.

because

STATEMENT-2 : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2+y^2=338$. *(2007 -3 marks)*

(a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True.

2. Consider $L_1: 2x+3y+p-3=0$

$L_2: 2x+3y+p+3=0$

where p is a real number, and $C: x^2+y^2+6x-10y+30=0$

STATEMENT - 1 : If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C

and

STATEMENT - 2 : If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C. *(2008)*

(a) Statement - 1 is True, Statement - 2 is True; Statement - 2 **is** a correct explanation for Statement - 1

(b) Statement - 1 is True, Statement - 2 is True; Statement - 2 **is NOT** a correct explanation for Statement - 1

(c) Statement - 1 is True, Statement - 2 is False

(d) Statement - 1 is False, Statement - 2 is True

I Integer Value Correct Type

1. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segement joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is *(2009)*

2. The straight line $2x-3y=1$ divides the circular region $x^2+y^2\le 6$ into two parts.

If $S=\left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\}$ then the number of points (s) in S lying inside the smaller part is *(2011)*

Section-B JEE Main / AIEEE

1. If the chord $y = mx + 1$ of the circle $x^2+y^2=1$ subtends an angle of measure 45° at the major segment of the circle then value of m is **[2002]**
   (a) $2 \pm \sqrt{2}$ (b) $-2 \pm \sqrt{2}$
   (c) $-1 \pm \sqrt{2}$ (d) none of these

2. The centres of a set of circles, each of radius 3, lie on the circle $x^2+y^2=25$. The locus of any point in the set is **[2002]**
   (a) $4 \le x^2+y^2 \le 64$ (b) $x^2+y^2 \le 25$
   (c) $x^2+y^2 \ge 25$ (d) $3 \le x^2+y^2 \le 9$

3. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2+y^2=9$ is **[2002]**
   (a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, -\sqrt{2}\right)$ (c) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \frac{3}{2}\right)$

4. The equation of a circle with origin as a centre and passing through equilateral triangle whose median is of length $3a$ is **[2002]**
   (a) $x^2+y^2=9a^2$ (b) $x^2+y^2=16a^2$
   (c) $x^2+y^2=4a^2$ (d) $x^2+y^2=a^2$

5. If the two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct point, then **[2003]**
   (a) $r > 2$ (b) $2 < r < 8$ (c) $r < 2$ (d) $r = 2$.

6. The lines $2x-3y=5$ and $3x-4y=7$ are diameters of a circle having area as 154 sq.units.Then the equation of the circle is **[2003]**
   (a) $x^2+y^2-2x+2y=62$ (b) $x^2+y^2+2x-2y=62$
   (c) $x^2+y^2+2x-2y=47$ (d) $x^2+y^2-2x+2y=47$

7. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=4$ orthogonally, then the locus of its centre is **[2004]**
   (a) $2ax-2by-(a^2+b^2+4)=0$
   (b) $2ax+2by-(a^2+b^2+4)=0$
   (c) $2ax-2by+(a^2+b^2+4)=0$
   (d) $2ax+2by+(a^2+b^2+4)=0$

8. A variable circle passes through the fixed point $A(p,q)$ and touches x-axis . The locus of the other end of the diameter through A is **[2004]**
   (a) $(y-q)^2=4px$ (b) $(x-q)^2=4py$
   (c) $(y-p)^2=4qx$ (d) $(x-p)^2=4qy$

9. If the lines $2x+3y+1=0$ and $3x-y-4=0$ lie along diameter of a circle of circumference 10π, then the equation of the circle is **[2004]**
   (a) $x^2+y^2+2x-2y-23=0$
   (b) $x^2+y^2-2x-2y-23=0$
   (c) $x^2+y^2+2x+2y-23=0$
   (d) $x^2+y^2-2x+2y-23=0$

10. Intercept on the line $y=x$ by the circle $x^2+y^2-2x=0$ is AB. Equation of the circle on AB as a diameter is **[2004]**
   (a) $x^2+y^2+x-y=0$
   (b) $x^2+y^2-x+y=0$
   (c) $x^2+y^2+x+y=0$
   (d) $x^2+y^2-x-y=0$

11. If the circles $x^2+y^2+2ax+cy+a=0$ and $x^2+y^2-3ax+dy-1=0$ intersect in two distinct points P and Q then the line $5x+by-a=0$ passes through P and Q for **[2005]**
   (a) exactly one value of a
   (b) no value of a
   (c) infinitely many values of a
   (d) exactly two values of a

12. A circle touches the x- axis and also touches the circle with centre at (0,3) and radius 2. The locus of the centre of the circle is **[2005]**
   (a) an ellipse (b) a circle
   (c) a hyperbola (d) a parabola

13. If a circle passes through the point (a, b) and cuts the circle $x^2+y^2=p^2$ orthogonally, then the equation of the locus of its centre is **[2005]**
   (a) $x^2+y^2-3ax-4by+(a^2+b^2-p^2)=0$
   (b) $2ax+2by-(a^2-b^2+p^2)=0$

(c) $x^2+y^2-2ax-3by+(a^2-b^2-p^2)=0$

(d) $2ax+2by-(a^2+b^2+p^2)=0$

14. If the pair of lines $ax^2+2(a+b)xy+by^2=0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then **[2005]**

(a) $3a^2-10ab+3b^2=0$ (b) $3a^2-2ab+3b^2=0$

(c) $3a^2+10ab+3b^2=0$ (d) $3a^2+2ab+3b^2=0$

15. If the lines $3x-4y-7=0$ and $2x-3y-5=0$ are two diameters of a circle of area 49π square units, the equation of the circle is **[2006]**

(a) $x^2+y^2+2x-2y-47=0$

(b) $x^2+y^2+2x-2y-62=0$

(c) $x^2+y^2-2x+2y-62=0$

(d) $x^2+y^2-2x+2y-47=0$

16. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its center is

(a) $x^2+y^2=\frac{3}{2}$ (b) $x^2+y^2=1$ **[2006]**

(c) $x^2+y^2=\frac{27}{4}$ (d) $x^2+y^2=\frac{9}{4}$

17. Consider a family of circles which are passing through the point (– 1, 1) and are tangent to x-axis. If (h, k) are the coordinate of the centre of the circles, then the set of values of k is given by the interval **[2007]**

(a) $-\frac{1}{2}\le k\le\frac{1}{2}$ (b) $k\le\frac{1}{2}$

(c) $0\le k\le\frac{1}{2}$ (d) $k\ge\frac{1}{2}$

18. The point diametrically opposite to the point $P(1, 0)$ on the circle $x^2+y^2+2x+4y-3=0$ is **[2008]**

(a) $(3,-4)$ (b) $(-3,4)$ (c) $(-3,-4)$ (d) $(3,4)$

19. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y=2$ is

(a) $(x-2)y'^2=25-(y-2)^2$

(b) $(y-2)y'^2=25-(y-2)^2$

(c) $(y-2)^2y'^2=25-(y-2)^2$

(d) $(x-2)^2y'^2=25-(y-2)^2$

20. If P and Q are the points of intersection of the circles $x^2+y^2+3x+7y+2p-5=0$ and $x^2+y^2+2x+2y-p^2=0$ then there is a circle passing through P, Q and (1, 1) for: **[2009]**

(a) all except one value of p

(b) all except two values of p

(c) exactly one value of p

(d) all values of p

21. The circle $x^2+y^2=4x+8y+5$ intersects the line $3x-4y=m$ at two distinct points if **[2010]**

(a) $-35<m<15$ (b) $15<m<65$

(c) $35<m<85$ (d) $-85<m<-35$

22. The two circles $x^2+y^2=ax$ and $x^2+y^2=c^2$ $(c>0)$ touch each other if **[2011]**

(a) $|a|=c$ (b) $a=2c$

(c) $|a|=2c$ (d) $2|a|=c$

23. The length of the diameter of the circle which touches the x-axis at the point (1,0) and passes through the point (2,3) is: **[2012]**

(a) $\frac{10}{3}$ (b) $\frac{3}{5}$ (c) $\frac{6}{5}$ (d) $\frac{5}{3}$

24. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point **[JEE M 2013]**

(a) (–5, 2) (b) (2, –5) (c) (5, –2) (d) (–2, 5)

25. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to **[JEE M 2014]**

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

26. Locus of the image of the point (2, 3) in the line $(2x-3y+4)+k(x-2y+3)=0$, $k\in\mathbf{R}$, is a : **[JEE M 2015]**

(a) circle of radius $\sqrt{2}$.

(b) circle of radius $\sqrt{3}$.

(c) straight line parallel to x-axis

(d) straight line parallel to y-axis

27. The number of common tangents to the circles $x^2+y^2-4x-6x-12=0$ and $x^2+y^2+6x+18y+26=0$, is : **[JEE M 2015]**

(a) 3 (b) 4 (c) 1 (d) 2

28. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on: **[JEE M 2016]**

(a) a hyperbola

(b) a parabola

(c) a circle

(d) an ellipse which is not a circle

29. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is: **[JEE M 2016]**

(a) 5

(b) 10

(c) $5\sqrt{2}$

(d) $5\sqrt{3}$

8 Circle

Section-A : JEE Advanced/ IIT-JEE

A

1. 1 2. (4, 2), (–2, –6) 3. $\frac{3}{4}$ 4. 8 sq. units 5. $x^2+y^2-x=0$ 6. $10x-3y-18=0$

7. $x^2+y^2+8x-6y+9=0$ 8. $\frac{192}{25}$ 9. $\left(-\frac{9}{5},\frac{12}{5}\right)$ or $\left(\frac{9}{5},-\frac{12}{5}\right)$ 10. $2\sqrt{3}$ sq. units

11. 2 12. $16x^2+16y^2-48x+16y+31=0$ 13. $x^2+y^2-x-y=0$ 14. 7 15. $\left(\frac{1}{2},\frac{1}{4}\right)$

B

1. T 2. T

C

1. (d)	2. (b)	3. (c)	4. (b)	5. (c)	6. (a)	7. (a)	8. (c)	9. (d)
10. (d)	11. (c)	12. (d)	13. (d)	14. (c)	15. (a)	16. (b)	17. (a)	18. (c)
19. (a)	20. (c)	21. (d)	22. (b)	23. (d)	24. (a)			

D

1. (a, c) 2. (b) 3. (a,b,c,d) 4. (a, c) 5. (b, c) 6. (a, c)

E

1. $x^2+y^2-18x-16y+120=0$

2. 75 sq. units 3. $x^2+y^2+2(10\pm\sqrt{54})x+55\pm\sqrt{54}=0$

5. $x^2+y^2+2ax+2py-b^2-q^2=0,\ \sqrt{a^2+p^2+b^2+q^2}$ 6. $x^2+y^2-10x-4y+4=0$

8. $k=1$ 10. $x^2+y^2+18x-2y+32=0$

11. $x^2+y^2+6x+2y-15=0$ and $x^2+y^2-10x-10y+25=0$ 12. $a^2>2b^2$ 13. $\left(2,\frac{23}{3}\right)$

14. $\left(\frac{14}{5},\frac{8}{5}\right)$, $y=0$ and $7y-24x+16=0$ 15. $a\in\,]-\infty,-2[\,\cup\,]2,\infty[$

19. $(x-4)^2+y^2=3^2$ and $\left(x+\frac{4}{3}\right)^2+y^2=\left(\frac{1}{3}\right)^2$; $y=\pm\frac{5}{\sqrt{39}}\left(x+\frac{4}{5}\right)$ 20. $3(3+\sqrt{10})$

21. ellipse 22. 5 23. $2x^2+2y^2-10x-5y+1=0$ 24. $\sqrt{5}$

G 1. (a) 2. (b) 3. (c) 4. (d) 5. (a) 6. (d) 7. (a) 8. (d)

H 1. (a) 2. (c)

I 1. 8 2. 2

Section-B : JEE Main/ AIEEE

1. (c)	2. (a)	3. (b)	4. (c)	5. (b)	6. (d)	7. (b)	8. (d)	9. (d)
10. (d)	11. (b)	12. (d)	13. (d)	14. (d)	15. (d)	16. (d)	17. (d)	18. (c)
19. (c)	20. (a)	21. (a)	22. (a)	23. (a)	24. (c)	25. (b)	26. (a)	27. (a)
28. (b)	29. (d)							

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. As P lies on a circle and A and B two points in the plane such that $\frac{PA}{PB} = k$

Then k can be any real number except 1 as otherwise P will lie on perpendicular bisector of AB which is a line.

2. For point of intersection of line

$$4x - 3y - 10 = 0 \quad \ldots(1)$$

and circle $x^2 + y^2 - 2x + 4y - 20 = 0$ ...(2)

Solving (1) and (2), we get

$$\left(\frac{3y+10}{4}\right)^2 + y^2 - 2\left(\frac{3y+10}{4}\right) + 4y - 20 = 0$$

$\Rightarrow y^2 + 4y - 12 = 0 \Rightarrow \quad y = 2, -6 \Rightarrow x = 4, -2$

$\therefore$ Points are (4, 2) and $(-2, -6)$

3. Let $3x - 4y + 4 = 0$ be the tangent at point A and $6x - 8y - 7 = 0$ be the tangent of point B of the circle.

As the two tangents parallel to each other

$\therefore$ AB should be the diameter of circle.

$\therefore$ AB = distance between parallel lines

$3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$

= distance between $6x - 8y + 8 = 0$ and

$6x - 8y - 7 = 0$

$$= \left|\frac{8+7}{\sqrt{36+64}}\right| = \frac{15}{10} = \frac{3}{2}$$

$\therefore$ radius of circle $= \frac{1}{2}(AB) = \frac{3}{4}$

4. 8 *sq*. units

KEY CONCEPT :

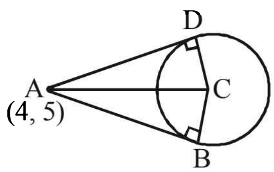
Length of tangent from a point (x_1, y_1) to a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is given by

$$\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

The equation of circle is,

$$x^2 + y^2 - 4x - 2y - 11 = 0$$

It's centre is (2, 1), radius $= \sqrt{4+1+11} = 4 = BC$



length of tangent from the *pt*. (4, 5) is

$$= \sqrt{16+25-16-10-11} = \sqrt{4} = 2 = AB$$

$\therefore$ Area of quad. $ABCD$

$$= 2\,(\text{Area of } \Delta ABC) = 2 \times \frac{1}{2} \times AB \times BC$$

$$= 2 \times \frac{1}{2} \times 2 \times 4 = 8 \text{ sq. units.}$$

5. The equation of given circle is

$(x-1)^2 + y^2 = 1$

or $x^2 + y^2 - 2x = 0$ ...(1)

KEY CONCEPT : We know that equation of chord of curve $S = 0$, whose mid point is (x_1, y_1) is given by $T = S_1$ where T is tangent to curve $S = 0$ at (x_1, y_1).

$\therefore$ If (x_1, y_1) is the mid point of chord of given circle (1), then equation of chord is

$$xx_1 + yy_1 - (x + x_1) = x_1^2 + y_1^2 - 2x_1$$

$$\Rightarrow (x_1 - 1)x + y_1 y + x_1 - x_1^2 - y_1^2 = 0$$

At it passes through origin, we get

$$x_1 - x_1^2 - y_1^2 = 0 \text{ or } x_1^2 + y_1^2 - x_1 = 0$$

$\therefore$ locus of (x_1, y_1) is $x^2 + y^2 - x = 0$

6. The equation of two circles are

$$x^2 + y^2 - \frac{2}{3}x + 4y - 3 = 0 \quad \ldots(1)$$

and $x^2 + y^2 + 6x + 2y - 15 = 0$ ...(2)

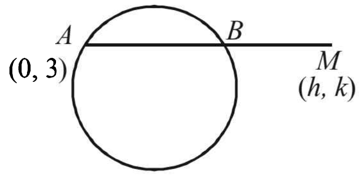
Now we know eq. of common chord of two circles $S_1 = 0$ and $S_2 = 0$ is $S_1 - S_2 = 0$

$$\Rightarrow 6x + \frac{2}{3}x + 2y - 4y - 15 + 3 = 0$$

$$\Rightarrow \frac{20x}{3} - 2y - 12 = 0 \Rightarrow 10x - 3y - 18 = 0$$

7. The equation of circle is,

$$x^2 + y^2 + 4x - 6y + 9 = 0 \quad \ldots(1)$$



$AM = 2AB$

$\Rightarrow AB = BM$

Let the co-ordinates of M be (h, k)

Then B is mid pt of AM

$$\therefore \quad B\left(\frac{0+h}{2}, \frac{3+k}{2}\right) = \left(\frac{h}{2}, \frac{k+3}{2}\right)$$

As B lies on circle (1),

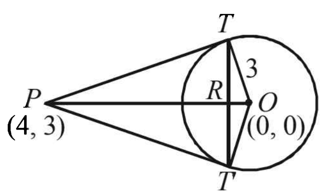
$$\therefore \left(\frac{h}{2}\right)^2 + \left(\frac{k+3}{2}\right)^2 + 4 \times \frac{h}{2} - 6\left(\frac{k+3}{2}\right) + 9 = 0$$

$\Rightarrow h^2 + k^2 + 8h - 6k + 9 = 0$

$\therefore$ locus of (h, k) is, $x^2 + y^2 + 8x - 6y + 9 = 0$

8. From $P(4, 3)$ two tangents PT and PT' are drawn to the circle $x^2 + y^2 = 9$ with O (0, 0) as centre and $r = 3$.

To find the area of $\Delta PTT'$.



Let R be the point of intersection of OP and TT'.
Then we can prove by simple geometry that OP is perpendicular bisector of TT'.
Equation of chord of contact TT' is $4x+3y=9$
Now, $OR =$ length of the perpendicular from O to TT' is

$$= \left| \frac{4\times 0+3\times 0-9}{\sqrt{4^2++3^2}} \right| = \frac{9}{5}$$

$OT =$ radius of circle $= 3$

$$\therefore \quad TR = \sqrt{OT^2 - OR^2} = \sqrt{9-\frac{81}{25}} = \frac{12}{5}$$

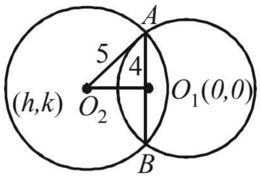
Again $OP = \sqrt{(4-0)^2+(3-0)^2} = 5$

$$\therefore \quad PR = OP - OR = 5 - \frac{9}{5} = \frac{16}{5}$$

Area of the triangle

$$PTT' = PR\times TR = \frac{16}{5}\times\frac{12}{5} = \frac{192}{25}$$

9. We have $C_1 : x^2+y^2=16$, Centre O_1 $(0, 0)$ radius $= 4$. C_2 is another circle with radius 5, let its centre O_2 be (h, k).



Now the common chord of circles C_1 and C_2 is of maximum length when chord is diameter of smaller circle C_1, and then it passes through centre O_1 of circle C_1. Given that slope of this chord is 3/4.

$\therefore$ Equation of AB is,

$$y = \frac{3}{4}x \Rightarrow 3x-4y=0 \qquad \ldots(1)$$

In right ΔAO_1O_2,

$$O_1O_2 = \sqrt{5^2-4^2} = 3$$

Also $O_1O_2 = \perp^{lar}$ distance from (h, k) to (1)

$$\Rightarrow \quad 3 = \left| \frac{3h-4k}{\sqrt{3^2+4^2}} \right| \Rightarrow \pm 3 = \frac{3h-4k}{5}$$

$$\Rightarrow \quad 3h-4k\pm 15 = 0 \qquad \ldots(2)$$

Again $AB \perp O_1O_2 \Rightarrow m_{AB}\times m_{O_1O_2} = -1$

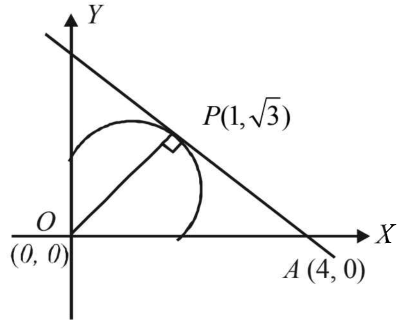
$$\Rightarrow \quad \frac{3}{4}\times\frac{k}{h} = -1 \Rightarrow 4h+3k=0 \qquad \ldots(3)$$

Solving, $3h-4k+15=0$ and $4h+3k=0$
We get $h=-9/5$, $k=12/5$
Again solving $3h-4k-15=0$ and $4h+3k=0$
We get $h=9/5$, $k=-12/5$

Thus the required centre is $\left(\frac{-9}{5}, \frac{12}{5}\right)$ or $\left(\frac{9}{5}, \frac{-12}{5}\right)$.

10. Tangent at $P(1, \sqrt{3})$ to the circle $x^2+y^2=4$ is

$$x\,.\,1 + y\,.\,\sqrt{3} = 4$$



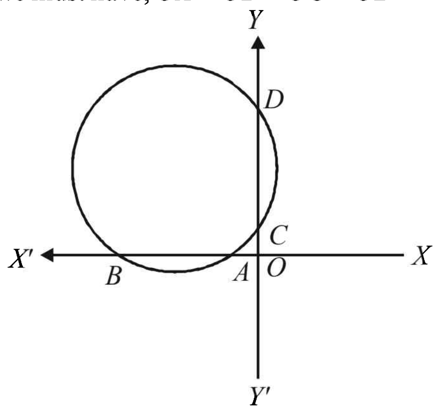
It meets x-axis at A (4, 0), $\therefore$ $OA = 4$
Also $OP =$ radius of circle $= 2$, $\therefore$ $PA = \sqrt{4^2-2^2} = \sqrt{12}$

$$\therefore \quad \text{Area of } \Delta OPA = \frac{1}{2}\times OP\times PA = \frac{1}{2}\times 2\times\sqrt{12}$$

$$= 2\sqrt{3} \text{ sq. units}$$

11. The given lines are $\lambda x - y + 1 = 0$ and $x-2y+3=0$ which meet x-axis at $A\left(-\frac{1}{\lambda}, 0\right)$ and B $(-3, 0)$ and y-axis at C (0, 1) and $D\left(0, \frac{3}{2}\right)$ respectively.

Then we must have, $OA \times OB = OC \times OD$



$$\Rightarrow \quad \left(-\frac{1}{\lambda}\right)(-3) = 1\times\frac{3}{2} \Rightarrow \lambda = 2$$

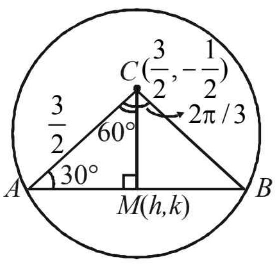
12. The given circle is,

$$4x^2+4y^2-12x+4y+1=0$$

or $\quad x^2+y^2-3x+y+\frac{1}{4}=0$ with centre $\left(\frac{3}{2}, -\frac{1}{2}\right)$

and $\quad r = \sqrt{\frac{9}{4}+\frac{1}{4}-\frac{1}{4}} = \frac{3}{2}$

Let M (h, k) be the mid pt. of the chord AB of the given circle, then $CM \perp AB$. $\angle ACB = 120°$.
In ΔACM,



$$\angle ACM = \frac{1}{2}\angle ACB = 60°$$

and $\angle A = 30°$

$$\therefore \quad \sin A = \frac{CM}{AC}$$

$$\sin 30° = \frac{\sqrt{(h-3/2)^2+(k+1/2)^2}}{3/2}$$

$\Rightarrow \left(\frac{3}{4}\right)^2 = \left(h-\frac{3}{2}\right)^2 + \left(k+\frac{1}{2}\right)^2$

$\Rightarrow 16h^2 + 16k^2 - 48h + 16k + 31 = 0$

$\therefore$ locus of (h, k) is $16x^2 + 16y^2 - 48x + 16y + 31 = 0$

13. Equation of any circle passing through the point of intersection of $x^2 + y^2 - 2x = 0$ and $y = x$ is

$x^2 + y^2 - 2x + \lambda(y - x) = 0$

or $x^2 + y^2 - (2+\lambda)x + \lambda y = 0$

Its centre is $\left(\frac{2+\lambda}{2}, \frac{-\lambda}{2}\right)$

For AB to be the diameter of the required circle, the centre must lie on AB. That is,

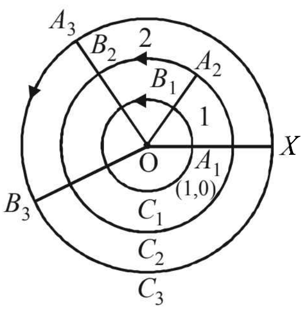
$\frac{2+\lambda}{2} = -\frac{\lambda}{2} \Rightarrow \lambda = -1$

Thus, equation of required circle is

$x^2 + y^2 - 2x - y + x = 0$

or $x^2 + y^2 - x - y = 0$

14.



The radius of circle C_1 is 1 cm, C_2 is 2 cm and soon.
It starts from A_1 (1, 0) on C_1, moves a distance of 1 cm on C_1 to come to B_1. The angle subtended by A_1B_1 at the centre will be $\frac{1}{r} = \theta$ radians, i.e. 1 radian.

From B_1 it moves along radius, OB_1 and comes to A_2 on circle C_2 of radius 2. From A_2 it moves on C_2 a distance 2 cm and comes to B_2. The angle subtended by A_2B_2 is again as before 1 radian. The total angle subtended at the centre is 2 radians. The process continues. In order to cross the x-axis again it must describe 2π radians i.e. $2.\frac{22}{7} = 6.7$ radians.

Hence it must be moving on circle C_7

$\therefore \quad n = 7$

15. Let (h, k) be any point on the given line

$\therefore \quad 2h + k = 4$ and chord of contact is $hx + ky = 1$

or $hx + (4-2h)y = 1$ or $(4y - 1) + h(x - 2y) = 0$

$P + \lambda Q = 0$. It passes through the intersection of $P = 0$ and $Q = 0$ i.e. $\left(\frac{1}{2}, \frac{1}{4}\right)$.

B. True/False

1. The circle passes through the points $A(1, \sqrt{3}), B(1, -\sqrt{3})$ and $C\ (3, -\sqrt{3})$.

Here line AB is parallel to y-axis and BC is parallel to x-axis, there $\angle ABC = 90°$

$\therefore$ AC is a diameter of circle.

$\therefore$ Eq. of circle is

$(x-1)(x-3) + (y-\sqrt{3})(y+\sqrt{3}) = 0$

$\Rightarrow x^2 + y^2 - 4x = 0 \qquad \ldots(1)$

Let us check the position of pt (5/2, 1) with. respect to the circle (1), we get $S_1 = \frac{25}{4} + 1 - 10 < 0$

$\therefore$ Point lies inside the circle.

$\therefore$ No tangent can be drawn to the given circle from point (5/2, 1).

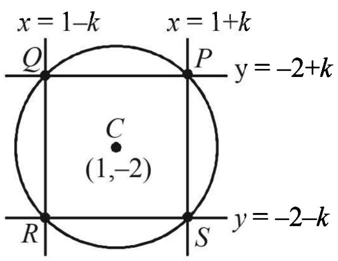
$\therefore$ Given statement is true.

2. The centre of the circle $x^2 + y^2 - 6x + 2y = 0$ is $(3, -1)$ which lies on the line $x + 3y = 0$

$\therefore$ The statement is true.

C. MCQs with ONE Correct Answer

1. **(d)** The given circle is $x^2 + y^2 - 2x + 4y + 3 = 0$. Centre $(1, -2)$. Lines through centre $(1, -2)$ and parallel to axes are $x = 1$ and $y = -2$.



Let the side of square be $2k$.
Then sides of square are $x = 1 - k$ and $x = 1 + k$ and $y = -2 - k$ and $y = -2 + k$

$\therefore$ Co-ordinates of P, Q, R, S are $(1+k, -2+k)$, $(1-k, -2+k)$, $(1-k, -2-k)$, $(1+k, -2-k)$ respectively.

Also $P(1+k, -2+k)$ lies on circle

$\therefore (1+k)^2 + (-2+k)^2 - 2(1+k) + 4(-2+k) + 3 = 0$

$\Rightarrow \ 2k^2 = 2 \ \Rightarrow \ k = 1$ or -1

If $k = 1$, $P(2, -1), Q(0, -1), R(0, -3), S(2, -3)$

If $k = -1$, $P(0, -3), Q(2, -3), R(2, -1), S(0, -1)$

2. **(b)** The circle through points of intersection of the two circles $x^2 + y^2 - 6 = 0$ and $x^2 + y^2 - 6x + 8 = 0$ is

$(x^2 + y^2 - 6) + \lambda(x^2 + y^2 - 6x + 8) = 0$

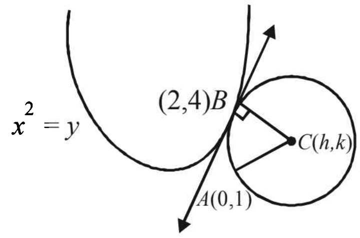
As it passes through (1, 1)

$(1 + 1 - 6) + 1(1 + 1 - 6 + 8) = 0 \Rightarrow \lambda = \frac{4}{4} = 1$

$\therefore$ The required circle is

$2x^2 + 2y^2 - 6x + 2 = 0$ or, $x^2 + y^2 - 3x + 1 = 0$

3. **(c)** Let $C(h, k)$ be the centre of circle touching $x^2 = y$ at B (2, 4). Then equation of common tangent at B is



$2.x = \frac{1}{2}(y+4)$ i.e., $4x - y = 4$

Radius is perpendicular to this tangent

$\therefore \quad 4\left(\frac{k-4}{h-2}\right) = -1 \Rightarrow 4k = 18 \quad \ldots(1)$

Also $AC = BC$

$\Rightarrow h^2 + (k-1)^2 = (h-2)^2 + (k-4)^2$

$\Rightarrow 4h + 6k = 19 \quad \ldots(2)$

Solving (1) and (2) we get the centre as $\left(-\frac{16}{5}, \frac{53}{10}\right)$.

4. (b) KEY CONCEPT

Circle through pts. of intersection of two circles $S_1 = 0$ and $S_2 = 0$ is $S_1 + \lambda S_2 = 0$

$\therefore$ Req. circle is,

$(x^2+y^2+13x-3y) + \lambda(x^2+y^2+2x-\frac{7}{2}y-\frac{25}{2}) = 0$

$\Rightarrow (1+\lambda)x^2 + (1+\lambda)y^2 + (13+2\lambda)x + \left(-3-\frac{7}{2}\lambda\right)y - \frac{25\lambda}{2} = 0$

As it passes through (1, 1)

$\therefore \quad 1+\lambda+1+\lambda+13+2\lambda-3-\frac{7\lambda}{2}-\frac{25\lambda}{2} = 0$

$\Rightarrow -12\lambda + 12 = 0 \Rightarrow \lambda = 1$

$\therefore$ Req. circle is,

$2x^2 + 2y^2 + 15x - \frac{13y}{2} - \frac{25}{2} = 0$

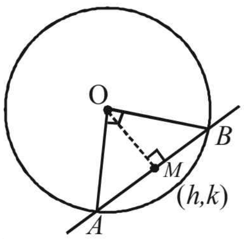
or $4x^2 + 4y^2 + 30x - 13y - 25 = 0$

5. (c) Let AB be the chord with its mid pt $M(h, k)$.

As $\angle AOB = 90°$

$\therefore \quad AB = \sqrt{2^2 + 2^2} = 2\sqrt{2}$.

$\therefore \quad AM = \sqrt{2}$



NOTE THIS STEP

By prop. of rt. Δ

$AM = MB = OM$

$\therefore \quad OM = \sqrt{2} \Rightarrow h^2 + k^2 = 2$

$\therefore$ locus of (h, k) is $x^2 + y^2 = 2$

6. (a) KEY CONCEPT

Two circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ are orthogonal iff $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

(a) Let the required circle be,

$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \ldots(1)$

As it passes through (a, b), we get,

$a^2 + b^2 + 2ag + 2bf + c = 0 \quad \ldots(2)$

Also (1) is orthogonal with the circle,

$x^2 + y^2 = k^2 \quad \ldots(3)$

For circle (1)

$g_1 = g, f_1 = f, c_1 = c$

For circle (3)

$g_2 = 0, f_2 = 0, c_2 = -k^2$

$\therefore$ By the condition of orthogonality,

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$

We get, $c = k^2$

Substituting this value of c in eq. (2), we get

$a^2 + b^2 + 2ga + 2fb + k^2 = 0$

$\therefore$ Locus of centre $(g - f)$ of the circle can be obtained by replacing g by $-x$ and f by $-y$ we get

$a^2 + b^2 - 2ax - 2by + k^2 = 0$

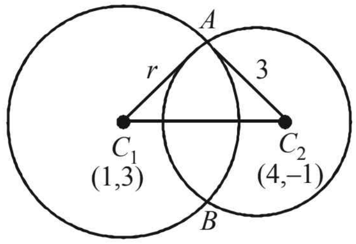
i.e. $2ax + 2by - (a^2 + b^2 + k^2) = 0$

7. (a) We have two circles $(x-1)^2 + (y-3)^2 = r^2$

Centre (1, 3), radius $= r$

and $x^2 + y^2 - 8x + 2y + 8 = 0$

Centre (4, – 1), radius $= \sqrt{16+1-8} = 3$



As the two circles intersect each other in two distinct points we should have

$C_1C_2 < r_1 + r_2$ and $C_1C_2 > |r_1 - r_2|$

$\Rightarrow C_1C_2 < r + 3$ and $C_1C_2 < |r_1 - r_2|$

$\Rightarrow \sqrt{9+16} < r + 3$ and $5 > |r - 3|$

$\Rightarrow 5 < r + 3 \quad \Rightarrow \quad |r-3| < 5$

$\Rightarrow r > 2 \ldots(i) \quad \Rightarrow \quad -5 < r - 3 < 5$

$\Rightarrow \quad -2 < r < 8 \ldots(ii)$

Combining (i) and (ii), we get

$2 < r < 8$

8. (c) As $2x - 3y - 5 = 0$ and $3x - 4y - 7 = 0$ are diameters of circles.

$\therefore$ Centre of circle is solution of these two eq.'s, i.e.

$\frac{x}{21-20} = \frac{y}{-15+14} = \frac{1}{-8+9}$

$\Rightarrow x = 1, y = -1$

$\therefore \quad C(1, -1)$

Also area of circle, $\pi r^2 = 154$

$\Rightarrow r^2 = \frac{154}{22} \times 7 = 49 \Rightarrow r = 7$

$\therefore$ Equation of required circle is

$(x-1)^2 + (y+1)^2 = 7^2 \Rightarrow x^2 + y^2 - 2x + 2y = 47$

9. (d) Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$.

As this circle passes through (0, 0) and (1, 0)

we get $c = 0, 1 + 2g = 0$

$\Rightarrow g = -\frac{1}{2}$

According to the question this circle touches the given circle $x^2 + y^2 = 9$

$\therefore$ 2 × radius of required circle = radius of given circle

$\Rightarrow 2\sqrt{g^2 + f^2} = 3 \Rightarrow g^2 + f^2 = \frac{9}{4}$

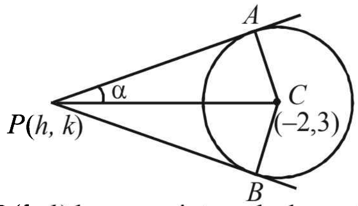
$\Rightarrow \frac{1}{4} + f^2 = \frac{9}{4} \Rightarrow f^2 = 2 \Rightarrow f = \pm\sqrt{2}$

$\therefore$ The centre is $\left(\frac{1}{2}, \sqrt{2}\right), \left(\frac{1}{2}, -\sqrt{2}\right)$.

10. (d) The given circle is $x^2 + y^2 - 6x + 14 = 0$, centre (3, 3), radius = 2
Let (*h, k*) be the centre of touching circle. Then radius of touching circle = *h* [as it touches *y*-axis also]
∴ Distance between centres of two circles = sum of the radii of two circles
$\Rightarrow \sqrt{(h-3)^2 + (k-3)^2} = 2 + h$
$\Rightarrow (h-3)^2 + (k-3)^2 = (2+h)^2$
$\Rightarrow k^2 - 10h - 6k + 14 = 0$
∴ locus of (*h, k*) is $y^2 - 10x - 6y + 14 = 0$

11. (c) Centres and radii of two circles are C_1 (5, 0); $3 = r_1$ and C_2 (0, 0); $r = r_2$
As circles intersect each other in two distinct points,
$|r_1 - r_2| < C_1C_2 < r_1 + r_2$
$\Rightarrow |r - 3| < 5 < r + 3 \Rightarrow 2 < r < 8$

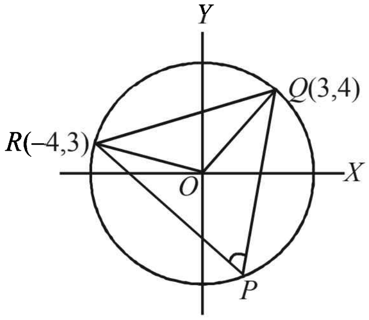
12. (d) Centre of the circle
$x^2 + y^2 + 4x - 6y + 9\sin^2\alpha + 13\cos^2\alpha = 0$
is *C* (– 2, 3) and its radius is
$\sqrt{2^2 + (-3)^2 - 9\sin^2\alpha - 13\cos^2\alpha}$
$= \sqrt{4 + 9 - 9\sin^2\alpha - 13\cos^2\alpha} = 2\sin\alpha$



Let *P* (*h, k*) be any point on the locus. The $\angle APC = \alpha$
Also $\angle PAC = \frac{\pi}{2}$
That is, triangle *APC* is a right triangle.
Thus, $\sin\alpha = \frac{AC}{PC} = \frac{2\sin\alpha}{\sqrt{(h+2)^2 + (k-3)^2}}$
$\Rightarrow \sqrt{(h+2)^2 + (k-3)^2} = 2$
$\Rightarrow (h+2)^2 + (k-3)^2 = 4$
or $h^2 + k^2 + 4h - 6k + 9 = 0$
Thus required equation of the locus is
$x^2 + y^2 + 4x - 6y + 9 = 0$

13. (d) The given equation of the circle is
$x^2 + y^2 - px - qy = 0, pq \neq 0$
Let the chord drawn from (*p, q*) is bisected by *x*-axis at point (x_1, 0).
Then equation of chord is
$x x_1 - \frac{p}{2}(x + x_1) - \frac{q}{2}(y + 0) = x_1^2 - p x_1$ (using $T = S_1$)
As it passes through (*p*, q), therefore,
$$p x_1 - \frac{p}{2}(p + x_1) - \frac{q^2}{2} = x_1^2 - p x_1$$
$$\Rightarrow x_1^2 - \frac{3}{2}p x_1 + \frac{p^2}{2} + \frac{q^2}{2} = 0$$
$$\Rightarrow 2x_1^2 - 3p x_1 + p^2 + q^2 = 0$$
As through (*p,q*) two distinct chords can be drawn.
∴ Roots of above equation be real and distinct, i.e., *D* > 0.
$\Rightarrow 9p^2 - 4 \times 2(p^2 + q^2) > 0$
$\Rightarrow p^2 > 8q^2$

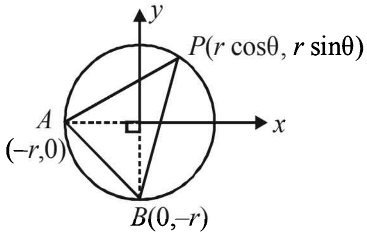
14. (c) *O* is the point at centre and *P* is the point at circumference. Therefore, angle *QOR* is double the angle *QPR*.



So, it sufficient to find the angle *QOR*. Now slope of *OQ* = 4/3
Slope of *OR* = – 3/4
Again $m_1 m_2 = -1$
Therefore, $\angle QOR = 90°$ which implies that $\angle QPR = 45°$.

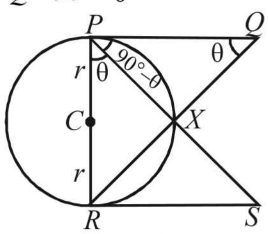
15. (a) $2g_1g_2 + 2f_1f_2 = c_1 + c_2$
(formula for orthogonal intersection of two cricles)
$\Rightarrow 2(1)(0) + 2(k)(k) = 6 + k$
$\Rightarrow 2k^2 - k - 6 = 0 \Rightarrow k = -3/2, 2$

16. (b) $x^2 + y^2 = r^2$ is a circle with centre at (0, 0) and radius *r* units.



Any arbitrary pt *P* on it is ($r\cos\theta$, $r\sin\theta$)
Choosing *A* and *B* as (– *r*, 0) and (0, – *r*).
[So that $\angle AOB = 90°$]
For locus of centriod of Δ*ABP*
$$\left(\frac{r\cos\theta - r}{3}, \frac{r\sin\theta - r}{3}\right) = (x, y)$$
$\Rightarrow r\cos\theta - r = 3x$
$r\sin\theta - r = 3y$
$\Rightarrow r\cos\theta = 3x + r$
$r\sin\theta = 3y + r$
Squaring and adding $(3x + r)^2 + (3y + r)^2 = r^2$ which is a circle.

17. (a) Let $\angle RPS = \theta$
$\angle XPQ = 90° - \theta$



∴ $\angle PQX = \theta$ (∵ $\angle PXQ = 90°$)
∴ $\Delta PRS \sim \Delta QPR$ (AA similarity)

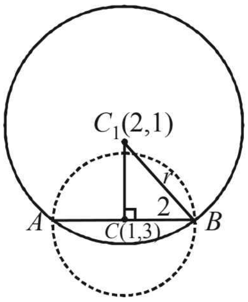
$\therefore \quad \frac{PR}{QP} = \frac{RS}{PR} \Rightarrow PR^2 = PQ \,.\, RS$

$\Rightarrow \quad PR = \sqrt{PQ.RS} \Rightarrow 2r = \sqrt{PQ.RS}$

18. (c) Line $5x - 2y + 6 = 0$ is intersected by tangent at P to circle $x^2 + y^2 + 6x + 6y - 2 = 0$ on y-axis at $Q(0, 3)$.
In other words tangent passes through (0, 3)
$\therefore \quad PQ$ = length of tangent to circle from (0, 3)
$= \sqrt{0+9+0+18-2}$
$= \sqrt{25} = 5$

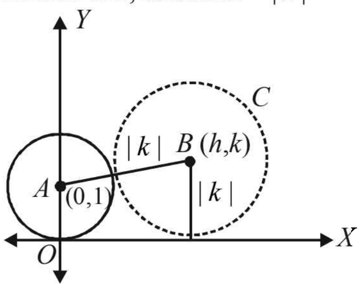
19. (a) $x^2 - 8x + 12 = 0 \Rightarrow (x-6)(x-2) = 0$
$y^2 - 14y + 45 = 0 \Rightarrow (y-5)(y-9) = 0$
Thus sides of square are
$x = 2, x = 6, y = 5, y = 9$
Then centre of circle inscribed in square will be
$\left(\frac{2+6}{2}, \frac{5+9}{2}\right) = (4, 7).$

20. (c) The given circle is $x^2 + y^2 - 2x - 6y + 6 = 0$ with centre $C(1, 3)$ and radius $= \sqrt{1+9-6} = 2$. Let AB be one of its diameter which is the chord of other circle with centre at $C_1(2, 1)$.
Then in ΔC_1CB,



$C_1B^2 = CC_1^2 + CB^2$
$r^2 = [(2-1)^2 + (1-3)^2] + (2)^2$
$\Rightarrow \quad r^2 = 1 + 4 + 4 \Rightarrow r^2 = 9 \Rightarrow \quad r = 3.$

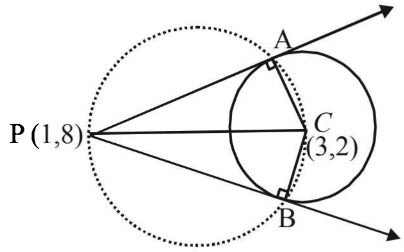
21. (d) Let the centre of circle C be (h, k). Then as this circle touches axis of x, its radius $= |k|$



Also it touches the given circle $x^2 + (y-1)^2 = 1$, centre (0, 1) radius 1, externally
Therefore, the distance between centres = sum of radii

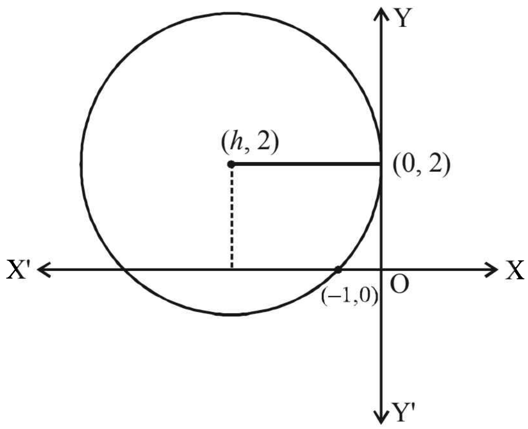
$\Rightarrow \quad \sqrt{(h-0)^2 + (k-1)^2} = 1 + |k|$
$\Rightarrow \quad h^2 + k^2 - 2k + 1 = (1 + |k|)^2$
$\Rightarrow \quad h^2 + k^2 - 2k + 1 = 1 + 2|k| + k^2$
$\Rightarrow \quad h^2 = 2k + 2|k|$
$\therefore \quad$ Locus of (h, k) is, $x^2 = 2y + 2|y|$
Now if $y > 0$, it becomes $x^2 = 4y$
and if $y \le 0$, it becomes $x = 0$
$\therefore \quad$ Combining the two, the required locus is
$\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

22. (b) Tangents PA and PB are drawn from the point P (1, 3) to circle $x^2 + y^2 - 6x - 4y - 11 = 0$ with centre C (3, 2)



Clearly the circumcircle of ΔPAB will pass through C and as $\angle A = 90°$, PC must be a diameter of the circle.
$\therefore \quad$ Equation of required circle is
$(x-1)(x-3) + (y-8)(y-2) = 0$
$\Rightarrow \quad x^2 + y^2 - 4x - 10y + 19 = 0$

23. (d) Let centre of the circle be (h, 2) then radius $= |h|$
$\therefore$ Equation of circle becomes $(x-h)^2 + (y-2)^2 = h^2$
As it passes through (–1, 0)



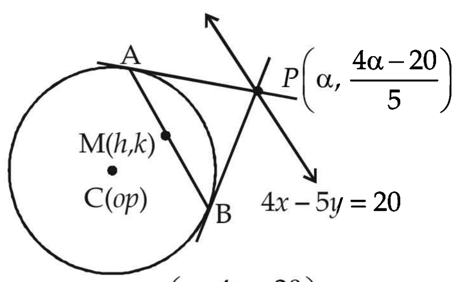
$\Rightarrow (-1-h)^2 + 4 = h^2 \Rightarrow h = \frac{-5}{2}$

$\therefore$ Centre $\left(\frac{-5}{2}, 2\right)$ and $r = \frac{5}{2}$

Distance of centre from (–4, 0) is $\frac{5}{2}$

$\therefore$ It lies on the circle.

24. (a) Any point P on line $4x - 5y = 20$ is $\left(\alpha, \frac{4\alpha - 20}{5}\right)$.
Equation of chord of contact AB to the circle $x^2 + y^2 = 9$



drawn from point $P\left(\alpha, \frac{4\alpha - 20}{5}\right)$ is

$x.\alpha + y.\left(\frac{4\alpha - 20}{5}\right) = 9 \quad$ ....(1)

Also the equation of chord AB whose mid point is (h, k) is

$$hx + ky = h^2 + k^2 \quad \text{....(2)}$$

$\because$ Equations (1) and (2) represent the same line, therefore

$$\frac{h}{\alpha} = \frac{k}{\frac{4\alpha - 20}{5}} = \frac{h^2 + k^2}{9}$$

$$\Rightarrow 5k\alpha = 4h\alpha - 20h \text{ and } 9h = \alpha\left(h^2 + k^2\right)$$

$$\Rightarrow \alpha = \frac{20h}{4h - 5k} \text{ and } \alpha = \frac{9h}{h^2 + k^2}$$

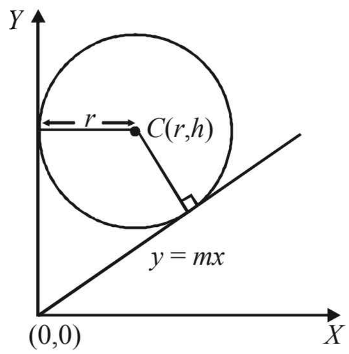
$$\Rightarrow \frac{20h}{4h - 5k} = \frac{9h}{h^2 + k^2} \Rightarrow 20(h^2 + k^2) = 9(4h - 5k)$$

$\therefore$ Locus of (h, k) is

$$20\left(x^2 + y^2\right) - 36x + 45y = 0$$

D. MCQs with ONE or MORE THAN ONE Correct

1. **(a, c)** The given circle is $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ with centre (r, h) and radius $= r$.
Clearly circle touches y-axis so one of its tangent is $x = 0$.



Let $y = mx$ be the other tangent through origin.
Then length of perpendicular from C (r, h) to $y = mx$ should be equal to r.

$$\therefore \left|\frac{mr - h}{\sqrt{m^2 + 1}}\right| = r$$

$$\Rightarrow m^2r^2 - 2mrh + h^2 = m^2r^2 + r^2$$

$$\Rightarrow m = \frac{h^2 - r^2}{2rh}$$

$\therefore$ Other tangent is $y = \frac{h^2 - r^2}{2rh}x$

or $(h^2 - r^2)x - 2rhy = 0$

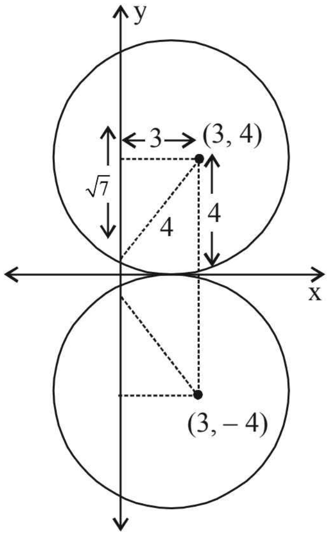
2. **(b)** $x^2 + y^2 = 4$ (given)
Centre $C_1 \equiv (0, 0)$ and $R_1 = 2$.
Also for circle $x^2 + y^2 - 6x - 8y - 24 = 0$
$C_2 \equiv (3, 4)$ and $R_2 = 7$.
Again $C_1 C_2 = 5 = R_2 - R_1$
Therefore, the given circles touch internally such that they can have just one common tangent at the point of contact.

3. **(a, b, c and d)**
Putting $y = c^2/x$ in $x^2 + y^2 = a^2$,
we obtain $x^2 + c^4/x^2 = a^2$

$$\Rightarrow x^4 - a^2x^2 + c^4 = 0 \quad \text{...(1)}$$

As x_1, x_2, x_3 and x_4 are roots of (1),
$\Rightarrow x_1 + x_2 + x_3 + x_4 = 0$ and $x_1 x_2 x_3 x_4 = c^4$
Similarly, forming equation in y, we get
$y_1 + y_2 + y_3 + y_4 = 0$ and $y_1 y_2 y_3 y_4 = c^4$.

4. **(a, c)** There can be two possibilites for the given circle as shown in the figure



$\therefore$ The equations of circles can be
$(x - 3)^2 + (y - 4)^2 = 4^2$
or $(x - 3)^2 + (y + 4)^2 = 4^2$
i.e. $x^2 + y^2 - 6x - 8y + 9 = 0$
or $x^2 + y^2 - 6x + 8y + 9 = 0$

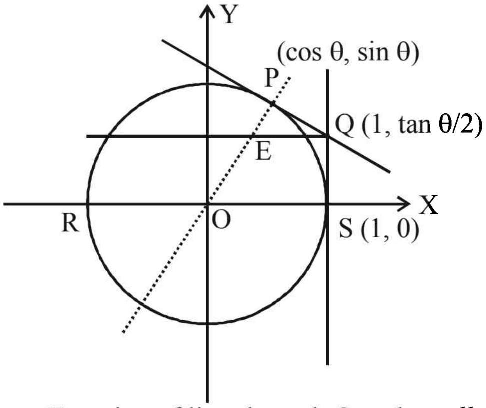
5. **(b, c)** Let the equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

It passes through (0, 1)
$\therefore 1 + 2f + c = 0$ ...(i)
This circle is orthogonal to $(x - 1)^2 + y^2 = 16$
i.e. $x^2 + y^2 - 2x - 15 = 0$
and $x^2 + y^2 - 1 = 0$
$\therefore$ We should have
$2g(-1) + 2f(0) = c - 15$
or $2g + c - 15 = 0$ ...(ii)
and $2g(0) + 2f(0) = c - 1$
or $c = 1$ ...(iii)
Solving (i), (ii) and (iii), we get
$c = 1, g = 7, f = -1$
$\therefore$ Required circle is
$x^2 + y^2 + 14x - 2y + 1 = 0$
With centre $(-7, 1)$ and radius $= 7$
$\therefore$ (b) and (c) are correct options.

6. **(a, c)** Circle : $x^2 + y^2 = 1$
Equation of tangent at $P(\cos\theta, \sin\theta)$
$x\cos\theta + y\sin\theta = 1$ ...(1)
Equation of normal at P
$y = x\tan\theta$ ...(2)
Equation of tangent at S is $x = 1$

$$\therefore Q\left(1, \frac{1 - \cos\theta}{\sin\theta}\right) = Q\left(1, \tan\frac{\theta}{2}\right)$$



$\therefore$ Equation of line through Q and parallel to RS is

$y = \tan\frac{\theta}{2}$

$\therefore$ Intersection point E of normal and $y = \tan\frac{\theta}{2}$

$\tan\frac{\theta}{2} = x\tan\theta \Rightarrow x = \frac{1-\tan^2\theta/2}{2}$

$\therefore$ Locus of E : $x = \frac{1-y^2}{2}$ or $y^2 = 1-2x$

It is satisfied by the points $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{3}, \frac{-1}{\sqrt{3}}\right)$

E. Subjective Problems

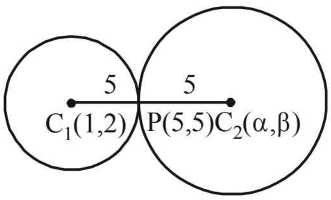
1. The given circle is

$x^2 + y^2 - 2x - 4y - 20 = 0$

whose centre is (1, 2) and radius = 5

Radius of required circle is also 5.

Let its centre be C_2 (α, β). Both the circles touch each other at $P(5, 5)$.



It is clear from figure that $P(5, 5)$ is the mid-point of C_1C_2. Therefore, we should have

$\frac{1+\alpha}{2} = 5$ and $\frac{2+\beta}{2} = 5 \Rightarrow \alpha = 9$ and $\beta = 8$

$\therefore$ Centre of required circle is (9, 8) and equation of required circle is $(x-9)^2 + (y-8)^2 = 5^2$

$\Rightarrow x^2 + y^2 - 18x - 16y + 120 = 0$

2. The eq. of circle is

$x^2 + y^2 - 2x - 4y - 20 = 0$

Centre (1, 2), radius $= \sqrt{1+4+20} = 5$

Using eq. of tangent at (x_1, y_1) of

$x^2 + y^2 + 2gx_1 + 2fy_1 + c = 0$ is

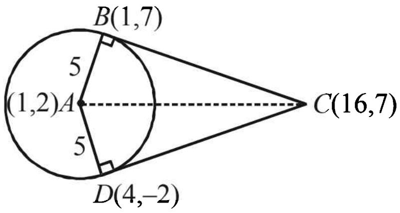
$xx_1 + yy_1 + g(x + x_1) f(y + y_1) + c = 0$

Eq. of tangent at (1, 7) is

$x \,.\, 1 + y \,.\, 7 - (x+1) - 2(y+7) - 20 = 0$

$\Rightarrow y - 7 = 0$ ...(1)

Similarly eq. of tangent at (4, – 2) is



$4x - 2y - (x+4) - 2(y-2) - 20 = 0$

$\Rightarrow 3x - 4y - 20 = 0$ ...(2)

For pt C, solving (1) and (2), we get

$x = 16, y = 7 \quad \therefore \quad C(16, 7).$

Now, clearly ar (quad $ABCD$) = 2 Ar ($rt\,\Delta ABC$)

$= 2 \times \frac{1}{2} \times AB \times BC = AB \times BC$

where AB = radius of circle = 5

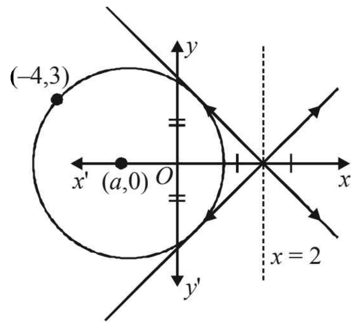
and BC = length of tangent from C to circle

$= \sqrt{16^2 + 7^2 - 32 - 28 - 20} = \sqrt{225} = 15$

$\therefore$ ar (quad $ABCD$) = 5 × 15 = 75 sq. units.

3. Given st. lines are $x + y = 2$

$x - y = 2$



As centre lies on ∠ bisector of given equations (lines) which are the lines $y = 0$ and $x = 2$.

$\therefore$ Centre lies on x axis or $x = 2$.

But as it passes through (– 4, 3), i.e., II quadrant.

$\therefore$ Centre must lie on x-axis

Let it be $(a, 0)$ then distance between $(a, 0)$ and $(-4, 3)$ is = length of ⊥ lar distance from $(a, 0)$ to $x + y - 2 = 0$

$\Rightarrow (a+4)^2 + (0-3)^2 = \left(\frac{a-2}{\sqrt{2}}\right)^2$

$\Rightarrow a^2 + 20a + 46 = 0 \Rightarrow a = -10 \pm \sqrt{54}$

$\therefore$ Equation of circle is

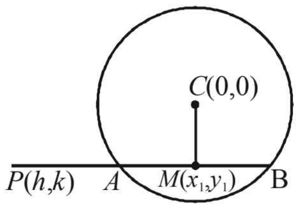
$\Rightarrow [x + (10 \pm \sqrt{54})]^2 + y^2 = [-(10 \pm \sqrt{54}) + 4]^2 + 3^2$

$\Rightarrow x^2 + y^2 + 2(10 \pm \sqrt{54})x + 8(10 \pm \sqrt{54}) - 25 = 0$

$\Rightarrow x^2 + y^2 + 2(10 \pm \sqrt{54})x + 55 \pm \sqrt{54} = 0.$

4. Equation of chord whose mid point is given is

$T = S_1$



[Consider (x_1, y_1) be mid pt. of AB]

$\Rightarrow \quad xx_1 + yy_1 - r^2 = x_1^2 + y_1^2 - r^2$

As it passes through (h, k),

$\therefore \quad hx_1 + ky_1 = x_1^2 + y_1^2$

$\therefore$ locus of (x_1, y_1) is,

$x^2 + y^2 = hx + ky$

5. Let the two points be

$A = (\alpha_1, \beta_1)$ and $B = (\alpha_2, \beta_2)$

Thus α_1, α_2 are roots of

$x^2 + 2ax - b^2 = 0$

$\therefore \quad \alpha_1 + \alpha_2 = -2a \quad \ldots(1)$

$\alpha_1 \alpha_2 = -b^2 \quad \ldots(2)$

β_1, β_2 are roots of $x^2 + 2px - q^2 = 0$

$\therefore \quad \beta_1 + \beta_2 = -2p \quad \ldots(3)$

$\beta_1\beta_2 = -q^2 \quad \ldots(4)$

Now equation of circle with AB as diameter is

$(x - \alpha_1)(x - \alpha_2) + (y - \beta_1)(y - \beta_2) = 0$

$\Rightarrow \quad x^2 - (\alpha_1 + \alpha_2)x + \alpha_1\alpha_2 + y^2 - (\beta_1 + \beta_2)y + \beta_1\beta_2 = 0$

$\Rightarrow \quad x^2 + 2ax - b^2 + y^2 + 2py - q^2 = 0$

[Using eq. (1), (2), (3) and (4)]

$\Rightarrow \quad x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$

Which is the equation of required circle, with its centre $(-a, -p)$ and radius $= \sqrt{a^2 + p^2 + b^2 + q^2}$.

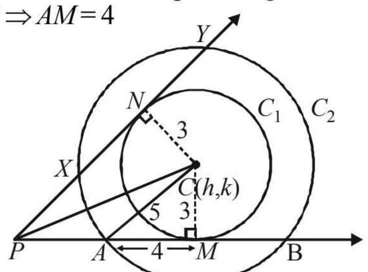
6. Let equation of tangent PAB be $5x + 12y - 10 = 0$ and that of PXY be

$5x - 12y - 40 = 0$

Now let centre of circles C_1 and C_2 be $C(h, k)$.

Let $CM \perp PAB$ then CM = radius of $C_1 = 3$

Also C_2 makes an intercept of length 8 units on $PAB \Rightarrow AM = 4$



Then in ΔAMC, we get

$AC = \sqrt{4^2 + 3^2} = 5$

$\therefore$ Radius of C_2 is $= 5$ units

Also, as $5x + 12y - 10 = 0 \quad \ldots(1)$

and $\quad 5x - 12y - 40 = 0 \quad \ldots(2)$

are tangents to C_1, length of perpendicular from C to $AB = 3$ units

$\therefore$ We get $\dfrac{5h + 12k - 10}{13} = \pm 3$

$\Rightarrow \quad 5h + 12k - 49 = 0 \quad \ldots(i)$

or $\quad 5h + 12k + 29 = 0 \quad \ldots(ii)$

Similarly, $\dfrac{5h - 12k - 40}{13} = \pm 3$

$\Rightarrow \quad 5h - 12k - 79 = 0 \quad \ldots(iii)$

or $\quad 5h - 12k - 1 = 0 \quad \ldots(iv)$

As C lies in first quadrant

$\therefore \quad h, k$ are $+ve$

$\therefore$ Eq. (ii) is not possible.

Solving (i) and (iii), we get

$h = 64/5, k = -5/4$

This is also not possible.

Now solving (i) and (iv), we get $h = 5, k = 2$.

Thus centre for C_2 is $(5, 2)$ and radius 5.

Hence, equation of C_2 is $(x - 5)^2 + (y - 2)^2 = 5^2$

$\Rightarrow \quad x^2 + y^2 - 10x - 4y + 4 = 0$

7. Let the equation of L_1 be

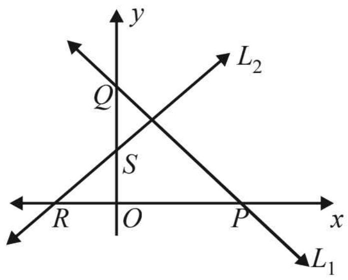
$x \cos \alpha + y \sin \alpha = p_1$

Then any line perpendicular to L_1 is

$x \sin \alpha - y \cos \alpha = p_2$, where p_2 is a variable.

Then L_1 meets x-axis at $P(p_1 \sec \alpha, 0)$ and y-axis at $Q(0, p_1 \operatorname{cosec} \alpha)$.

Similarly L_2 meets x-axis at $R(p_2 \operatorname{cosec} \alpha, 0)$ and y-axis at $S(0, -p_2 \sec \alpha)$.



Now equation of PS is,

$$\frac{x}{p_1 \sec \alpha} + \frac{y}{-p_2 \sec \alpha} = 1 \Rightarrow \frac{x}{p_1} - \frac{y}{p_2} = \sec \alpha \quad \ldots(1)$$

Similarly, equation of QR is,

$$\Rightarrow \quad \frac{x}{p_2 \operatorname{cosec} \alpha} + \frac{y}{p_1 \operatorname{cosec} \alpha} = 1$$

$$\Rightarrow \quad \frac{x}{p_2} + \frac{y}{p_1} = \operatorname{cosec} \alpha \quad \ldots(2)$$

Locus of point of intersection of PS and QR can be obtained by eliminating the variable p_2 from (1) and (2)

i.e. $\left(\dfrac{x}{p_1} - \sec \alpha\right)\dfrac{x}{y} + \dfrac{y}{p_1} = \operatorname{cosec} \alpha$

[Substituting the value of $\dfrac{1}{p_2}$ from (1) in (2)]

$\Rightarrow \quad (x - p_1 \sec \alpha)x + y^2 = p_1 y \operatorname{cosec} \alpha$

$\Rightarrow \quad x^2 + y^2 - p_1 x \sec \alpha - p_1 y \operatorname{cosec} \alpha = 0$

which is a circle through origin.

8. The given circle is

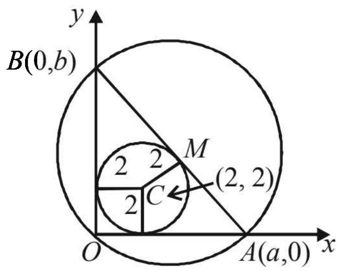
$x^2 + y^2 - 4x - 4y + 4 = 0.$

This can be re-written as

$(x - 2)^2 + (y - 2)^2 = 4$

which has centre $C(2, 2)$ and radius 2.

Let the eq. of third side AB of ΔOAB is $\dfrac{x}{a} + \dfrac{y}{b} = 1$ such that $A(a, 0)$ and $B(0, b)$



Length of perpendicular form (2, 2) on AB = radius = $CM = 2$

$$\therefore \quad \frac{\left|\frac{2}{a}+\frac{2}{b}-1\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=2$$

Since (2, 2) and origin lie on same side of AB

$$\therefore \quad \frac{-\left(\frac{2}{a}+\frac{2}{b}-1\right)}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}=2$$

$$\Rightarrow \quad \frac{2}{a}+\frac{2}{b}-1=-2\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} \qquad \ldots(1)$$

Since $\angle AOB = \pi/2$.
Hence, AB is the diameter of the circle passing through ΔOAB, mid point of AB is the centre of the circle i.e. $(a/2, b/2)$

Let centre be $(h, k) \equiv \left(\frac{a}{2}, \frac{b}{2}\right)$

then $a = 2h$, $b = 2k$.
Substituting the values of a and b in (1), we get

$$\frac{2}{2h}+\frac{2}{2k}-1=-2\sqrt{\frac{1}{4h^2}+\frac{1}{4k^2}}$$

$$\Rightarrow \quad \frac{1}{h}+\frac{1}{k}-1=-\sqrt{\frac{1}{h^2}+\frac{1}{k^2}} \Rightarrow h+k-hk+\sqrt{h^2+k^2}=0$$

$\therefore$ Locus of $M(h, k)$ is,

$$x+y-xy+\sqrt{x^2+y^2}=0 \qquad \ldots(2)$$

Comparing it with given equation of locus of circumcentre of Δ i.e.

$$x+y-xy+k\sqrt{x^2+y^2}=0 \qquad \ldots(3)$$

We get, $k = 1$

9. Given that $\left(m_i, \frac{1}{m_i}\right)$, $m_i > 0$, i = 1, 2, 3, 4 are four distinct points on a circle.
Let the equation of circle be $x^2+y^2+2gx+2fy+c=0$

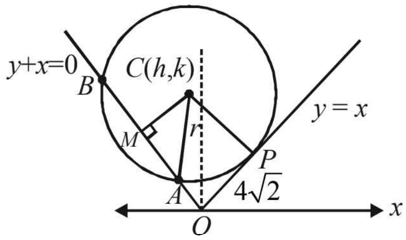
As the point $\left(m, \frac{1}{m}\right)$ lies on it, therefore, we have

$$m^2+\frac{1}{m^2}+2gm+\frac{2f}{m}+c=0$$

$$\Rightarrow \quad m^4+2gm^3+cm^2+2fm+1=0$$

Since m_1, m_2, m_3, m_4 are roots of this equation, therefore product of roots = 1 $\Rightarrow m_1m_2m_3m_4 = 1$

10. Let AB be the length of chord intercepted by circle on $y + x = 0$
Let CM be perpendicular to AB from centre $C(h, k)$.



Also $y - x = 0$ and $y + x = 0$ are perpendicular to each other.
$\therefore$ $OPCM$ is rectangle.
$\therefore$ $CM = OP = 4\sqrt{2}$.
Let r be the radius of cirlce.

Also $AM = \frac{1}{2}AB = \frac{1}{2}\times 6\sqrt{2} = 3\sqrt{2}$

$\therefore$ In ΔCAM, $AC^2 = AM^2 + MC^2$

$\Rightarrow$ $r^2 = (3\sqrt{2})^2 + (4\sqrt{2})^2 \Rightarrow r^2 = (5\sqrt{2})^2$

$\Rightarrow$ $r = 5\sqrt{2}$

Again $y = x$ is tangent to the circle at P
$\therefore$ $CP = r$

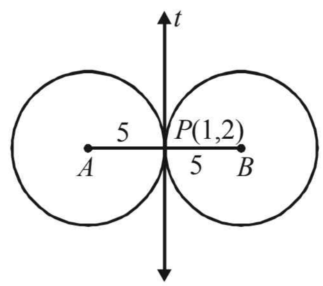
$$\Rightarrow \quad \left|\frac{h-k}{\sqrt{2}}\right| = 5\sqrt{2} \Rightarrow h-k=\pm 10 \qquad \ldots(1)$$

Also $CM = 4\sqrt{2}$

$$\Rightarrow \quad \left|\frac{h+k}{\sqrt{2}}\right| = 4\sqrt{2} \Rightarrow h+k=\pm 8 \qquad \ldots(2)$$

Solving four sets of eq's given by (1) and (2), we get the possible centres as
$(9, -1), (1, -9), (-1, 9), (-9, 1)$
$\therefore$ Possible circles are
$(x-9)^2+(y+1)^2-50=0$
$(x-1)^2+(y+9)^2-50=0$
$(x+1)^2+(y-9)^2-50=0$
$(x+9)^2+(y-1)^2-50=0$
But the pt $(-10, 2)$ lies inside the circle.
$\therefore$ $S_1 < 0$ which is satisfied only for $(x+9)^2+(y-1)^2-50=0$
$\therefore$ The required eq. of circle is
$x^2+y^2+18x-2y+32=0$.

11. Let t be the common tangent given by
$4x + 3y = 10 \qquad \ldots(1)$
Common pt of contact being $P(1, 2)$
Let A and B be the centres of the circles, required. Clearly, AB is the line perpendicular to t and passing through $P(1, 2)$.



Therefore eq. of AB is

$$\frac{x-1}{4/5}=\frac{x-2}{3/5}=r\quad\left[\begin{array}{l}\text{As slope of } t \text{ is}=-4/3\\ \therefore \text{ slope of } AB \text{ is}=3/4=\tan\theta\\ \therefore \cos\theta=4/5;\sin\theta=3/5\end{array}\right.$$

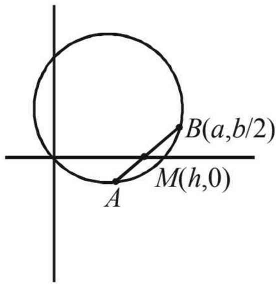
For pt A, $r=-5$ and for pt B, $r=5$, we get

$$\frac{x-1}{4/5}=\frac{y-2}{3/5}=-5,5\left(\begin{array}{l}\text{radius of each circle}\\ \text{being } 5,\ AP=PB=5\end{array}\right)$$

$\Rightarrow$ For pt A $x=-4+1, y=-3+2$
and For pt B $x=4+1, y=3+2$
$\therefore$ $A(-3,-1)\ B(5,5)$.
$\therefore$ Eq.'s of required circles are
$(x+3)^2+(y+1)^2=5^2$
and $(x-5)^2+(y-5)^2=5^2$

$$\Rightarrow \left.\begin{array}{r}x^2+y^2+6x+2y-15=0\\ \text{and } x^2+y^2-10x-10y+25=0\end{array}\right\}$$

12. The given circle is
$2x(x-a)+y(2y-b)=0\ (a, b\neq 0)$
$\Rightarrow$ $2x^2+2y^2-2ax-by=0$ ....(1)
Let us consider the chord of this circle which passes through the pt $\left(a,\frac{b}{2}\right)$ and whose mid pt. lies on x-axis.



Let $(h, 0)$ be the mid point of the chord, then eq. of chord can be obtained by $T=S_1$

i.e., $2xh+2y.0-a(x+h)-\frac{b}{2}(y+0)=2h^2-2ah$

$\Rightarrow$ $(2h-a)x-\frac{b}{2}y+ah-2h^2=0$

This chord passes through $\left(a,\frac{b}{2}\right)$, therefore

$(2h-a)a-\frac{b}{2}.\frac{b}{2}+ah-2h^2=0$

$\Rightarrow$ $8h^2-12ah+(4a^2+b^2)=0$
As given in question, two such chords are there, so we should have two real and distinct values of h from the above quadratic in h, for which
$D>0$
$\Rightarrow$ $(12a)^2-4\times 8\times(4a^2+b^2)>0$
$\Rightarrow$ $a^2>2b^2$

13. Let the family of circles, passing through $A(3, 7)$ and $B(6, 5)$, be
$x^2+y^2+2gx+2fy+c=0$
As it passes through $(3, 7)$
$\therefore$ $9+49+6g+14f+c=0$
or, $6g+14f+c+58=0$ ...(1)
As it passes through $(6, 5)$
$\therefore$ $36+25+12g+10f+c=0$
$12g+10f+c+61=0$ ...(2)
$(2)-(1)$ gives,

$6g-4f+3=0 \Rightarrow g=\frac{4f-3}{6}$

Substituting the value of g in equation (1), we get
$4f-3+14f+c+58=0$
$\Rightarrow$ $18f+55+c=0 \Rightarrow c=-18f-55$
Thus the family is

$$x^2+y^2+\left(\frac{4f-3}{3}\right)x+2fy-(18f+55)=0$$

Members of this family are cut by the circle
$x^2+y^2-4x-6y-3=0$
$\therefore$ Equation of family of chords of intersection of above two circles is
$S_1-S_2=0$

$\Rightarrow$ $\left(\frac{4f-3}{3}+4\right)x+(2f+6)y-18f+52)=0$

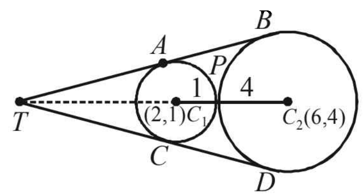
which can be written as

$$(3x+6y-52)+f\left(\frac{4}{3}x+2y-18\right)=0$$

which represents the family of lines passing through the pt. of intersection of the lines
$3x+6y-52=0$ and $4x+6y-54=0$
Solving which we get $x=2$ and $y=23/3$.

Thus the required pt. of intersection is $\left(2,\frac{23}{3}\right)$

14. The given circles are
$x^2+y^2-4x-2y=-4$
and $x^2+y^2-12x-8y=-36$
i.e., $x^2+y^2-4x-2y+4=0$ ... (1)
$x^2+y^2-12x-8y+36=0$ .... (2)
with centres $C_1(2, 1)$ and $C_2(6,4)$ and radii 1 and 4 respectively.
Also $C_1C_2=5$
As $r_1+r_2=C_1C_2$
$\Rightarrow$ Two circles touch each other externally, at P.



Clearly, P divides C_1C_2 in the ratio 1 : 4
$\therefore$ Co-ordinates of P are

$$\left(\frac{1\times 6+4\times 2}{1+4},\frac{1\times 4+4\times 1}{4+1}\right)=\left(\frac{14}{5},\frac{8}{5}\right)$$

Let AB and CD be two common tangents of given circles, meeting each other at T. Then T divides C_1C_2 externally in the ratio 1 : 4.
KEY CONCEPT : [As the direct common tangents of two circles pass through a pt. which divides the line segment joining the centres of two circles externally in the ratio of their radii.]

Hence, $T\equiv\left(\frac{1\times 6-4\times 2}{1-4},\frac{1\times 4-4\times 1}{1-4}\right)=\left(\frac{2}{3},0\right)$

Let m be the slope of the tangent, then equation of tangent through (2/3, 0) is

$$y-0=m\left(x-\frac{2}{3}\right) \Rightarrow y-mx+\frac{2}{3}m=0$$

Now, length of perpendicular from (2, 1), to the above tangent is radius of the circle

$$\therefore \left|\frac{1-2m+\frac{2}{3}m}{\sqrt{m^2+1}}\right|=1$$

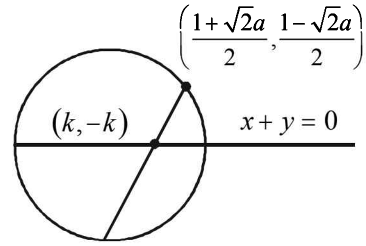
$\Rightarrow (3-4m)^2=9(m^2+1) \Rightarrow 9-24m+16m^2=9m^2+9$

$\Rightarrow 7m^2-24m=0 \Rightarrow m=0, \frac{24}{7}$

Thus the equations of the tangents are $y=0$ and $7y-24x+16=0$.

15. Let the given point be

$(p,\bar{p})=\left(\frac{1+\sqrt{2}a}{2},\frac{1-\sqrt{2}a}{2}\right)$ and the equation of the circle becomes $x^2+y^2-px-\bar{p}y=0$



Since the chord is bisected by the line $x+y=0$, its mid-point can be chosen as $(k,-k)$. Hence the equation of the chord by $T=S_1$ is

$$kx-ky-\frac{p}{2}(x+k)-\frac{\bar{p}}{2}(y-k)=k^2+k^2-pk+\bar{p}k$$

It passes through $A(p,\bar{p})$

$\therefore \quad kp-k\bar{p}-\frac{p}{2}(p+k)-\frac{\bar{p}}{2}(\bar{p}-k)=2k^2-pk+\bar{p}k$

or $\quad 3k(p-\bar{p})=4k^2+(p^2+\bar{p}^2) \quad \ldots(1)$

Put $p-\bar{p}=a\sqrt{2}, p^2-\bar{p}^2=2.\frac{(1+2a^2)}{4}=\frac{1+2a^2}{2} \quad \ldots(2)$

Hence, from (1) by the help of (2), we get

$$4k^2-3\sqrt{2}ak+\frac{1}{2}(1+2a^2)=0 \quad \ldots(3)$$

Since, there are two chords which are bisected by $x+y=0$, we must have two real values of k from (3)

$\therefore \quad \Delta>0$

or $\quad 18a^2-8(1+2a^2)>0$

or, $\quad a^2-4>0$

or, $\quad (a+2)(a-2)>0$

$\therefore \quad a<-2$ or >2

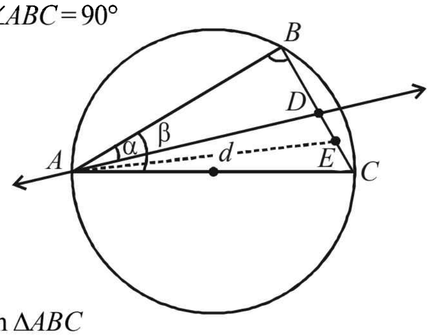
$\therefore \quad a\in(-\infty,-2)\cup(2,\infty)$

or $\quad a\in]-\infty,-2[\cup]2,\infty[$

16. Let r be the radius of circle, then $AC=2r$

Since, AC is the diameter

$\therefore \quad \angle ABC=90°$



$\therefore$ In ΔABC

$BC=2r\sin\beta, AB=2r\cos\beta$

In rt $\angle$ed ΔABC

$BD=AB\tan\alpha=2r\cos\beta\tan\alpha$

$AD=AB\sec\alpha=2r\cos\beta\sec\alpha$

$\therefore \quad DC=BC-BD=2r\sin\beta-2r\cos\beta\tan\alpha$

Now since E is the mid point of DC

$$\therefore \quad DE=\frac{DC}{2}=\frac{2r\sin\beta-2r\cos\beta\tan\alpha}{2}$$

$\Rightarrow \quad DE=r\sin\beta-r\cos\beta\tan\alpha$

Now in ΔADC, AE is the median

$\therefore \quad 2(AE^2+DE^2)=AD^2+AC^2$

$\Rightarrow \quad 2[d^2+r^2(\sin\beta-\cos\beta\tan\alpha)^2]$
$=4r^2\cos^2\beta\sec^2\alpha+4r^2$

$$\Rightarrow \quad r^2=\frac{d^2\cos^2\alpha}{\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta\cos(\beta-\alpha)}$$

$\Rightarrow$ Area of circle,

$$\pi r^2=\frac{\pi d^2\cos^2\alpha}{\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta\cos(\beta-\alpha)}$$

17. Given C is the circle with centre at $(0,\sqrt{2})$ and radius r (say)

then $C\equiv x^2+(y-\sqrt{2})^2=r^2$

$\Rightarrow \quad (y-\sqrt{2})^2=(r^2-x^2) \Rightarrow y-\sqrt{2}=\pm\sqrt{r^2-x^2}$

$\Rightarrow \quad y=\sqrt{2}\pm\sqrt{r^2-x^2} \quad \ldots(1)$

The only rational value which y can have is 0. Suppose the possible value of x for which y is 0 is x_1. Certainly $-x_1$ will also give the value of y as 0 (from (1)). Thus, at the most, there are two rational pts which satisfy the eq^n of C.

18. Let $P(h,k)$ be on C_2

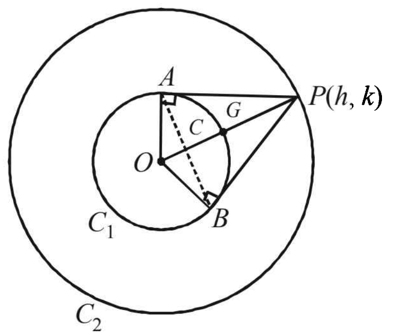
$\therefore \quad h^2+k^2=4r^2$

Chord of contact of P w.r.t. C_1 is

$hx+ky=r^2$

It intersects C_1,

$x^2+y^2=a^2$ in A and B.



Eliminating y, we get,

$$x^2+\left(\frac{r^2-hx}{k}\right)^2=r^2$$

or, $x^2(h^2+k^2)-2r^2hx+r^4-r^2k^2=0$
or, $x^2.4r^2-2r^2hx+r^2(r^2-k^2)=0$

$$\therefore\quad x_1+x_2=\frac{2r^2h}{4r^2}=\frac{h}{2}, y_1+y_2=\frac{k}{2}$$

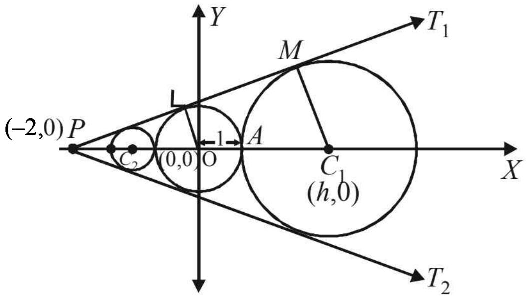
If (x, y) be the centroid of ΔPAB, then

$$3x=x_1+x_2+h=\frac{h}{2}+h=\frac{3h}{2}$$

$$\therefore\quad x=\frac{h}{2} \text{ or } h=2x \text{ and similarly } k=2y$$

Putting in (1) we get
$4x^2+4y^2=4r^2$
$\therefore$ Locus is $x^2+y^2=r^2$ i.e., C_1

19. The given circle is $x^2+y^2=1$ ...(1)
Centre $O(0, 0)$ radius $=1$
Let T_1 and T_2 be the tangents drawn from $(-2, 0)$ to the circle (1).



Let m be the slope of tangent then equations of tangents are
$y-0=m(x+2)$
or, $mx-y+2m=0$ ...(2)
As it is tangent to circle (1) length of $\perp$ lar from (0, 0) to (2) = radius of (1)

$$\Rightarrow \left|\frac{2m}{\sqrt{m^2+1}}\right|=1 \Rightarrow 4m^2=m^2+1 \Rightarrow m=\pm 1/\sqrt{3}$$

$\therefore$ The two tangents are $x+\sqrt{3}y+2=0\,(T_1)$ and

$$x-\sqrt{3}y+2=0\,(T_2)$$

Now any other circle touching (1) and T_1, T_2 is such that its centre lies on x-axis.
Let $(h, 0)$ be the centre of such circle, then from fig.
$OC_1=OA+AC_1 \Rightarrow |h|=1+|AC_1|$
But $AC_1=\perp$ lar distance of $(h, 0)$ to tangent

$$\Rightarrow |h|=1+\left|\frac{h+2}{2}\right| \Rightarrow |h|-1=\left|\frac{h+2}{2}\right|$$

Squaring,

$$h^2-2|h|+1=\frac{h^2+4h+4}{4}$$

$\Rightarrow 4h^2\pm 8h+4=h^2+4h+4$
'+' $\Rightarrow 3h^2=-4h \Rightarrow h=-4/3$
'–' $\Rightarrow 3h^2=12h \Rightarrow h=4$

Thus centres of circles are (4, 0), $\left(-\frac{4}{3}, 0\right)$.

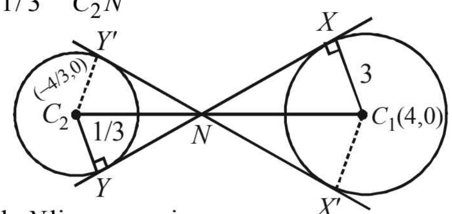
$\therefore$ Radius of circle with centre (4, 0) is $=4-1=3$ and radius of circle with centre $\left(\frac{-4}{3}, 0\right)$ is $=\frac{4}{3}-1=\frac{1}{3}$

$\therefore$ The two possible circles are
$(x-4)^2+y^2=3^2$ ...(3)

And $\left(x+\frac{4}{3}\right)+y^2=\left(\frac{1}{3}\right)^2$ ...(4)

Now, common tangents of (1) and (3). Since (1) and (3) are two touching circles they have three common tangents T_1, T_2 and $x=1$ (clear from fig.)
Similarly common tangents of (1) and (4) are T_1, T_2 and $x=-1$.
For the circles (3) and (4) there will be four common tangents of which two are direct common tangents.
XY and $x'y'$ and two are indirect common tangents. Let us find two common indirect tangents. We know that
In two similar Δ's C_1XN and C_2YN

$$\frac{3}{1/3}=\frac{C_1N}{C_2N} \Rightarrow N \text{ divides } C_1C_2 \text{ in the ratio } 9:1.$$



Clearly N lies on x-axis.

$$\therefore\quad N=\left(\frac{9\times(-4/3)+1\times 4}{10}, 0\right)=\left(\frac{-4}{5}, 0\right)$$

Any line through N is

$$y=m\left(x+\frac{4}{5}\right) \quad \text{or} \quad 5mx-5y+4m=0$$

If it is tangent to (3) then

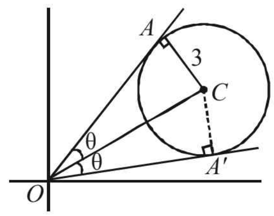
$$\left|\frac{20m+4m}{\sqrt{25m^2+25}}\right|=3$$

$\Rightarrow 24m=15\sqrt{m^2+1} \Rightarrow 64m^2=25m^2+25$
$\Rightarrow 39m^2=25 \Rightarrow m=\pm 5/\sqrt{39}$
$\therefore$ Required tangents are

$$y=\pm\frac{5}{\sqrt{39}}\left(x+\frac{4}{5}\right).$$

20. The equation $2x^2-3xy+y^2=0$ represents pair of tangents OA and OA'.
Let angle between these to tangents be 2θ.

$$\text{Then, } \tan 2\theta=\frac{2\sqrt{\left(\frac{-3}{2}\right)^2-2\times 1}}{2+1}$$



$$\left[\text{Using } \tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}\right]$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{1}{3} \Rightarrow \tan^2\theta + 6\tan\theta - 1 = 0$$

$$\tan\theta = \frac{-6 \pm \sqrt{36+4}}{2} = -3 \pm \sqrt{10}$$

As θ is acute $\tan\theta = \sqrt{10} - 3$

Now we know that line joining the pt through which tangents are drawn to the centre bisects the angle between the tangents,

$\therefore \quad \angle AOC = \angle A'OAC = \theta$

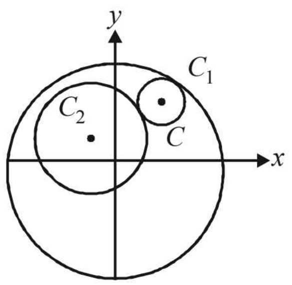
In ΔAOC,

$$\tan\theta = \frac{3}{OA} \qquad \Rightarrow \text{OA} = \frac{3}{\sqrt{10}-3} \times \frac{\sqrt{10}+3}{\sqrt{10}+3}$$

$\therefore \quad OA = 3(3+\sqrt{10})$.

21. Let equation of C_1 be $x^2 + y^2 = r_1^2$ and of C_2 be

$$(x-a)^2 + (y-b)^2 = r_2^2$$



Let centre of C be (h, k) and radius be r, then by the given conditions.

$$\sqrt{(h-a)^2 + (k-b)^2} = r + r_2 \text{ and } \sqrt{h^2 + k^2} = r_1 - r$$

$$\Rightarrow \sqrt{(h-a)^2 + (k-b)^2} + \sqrt{h^2 + k^2} = r_1 + r_2$$

Required locus is

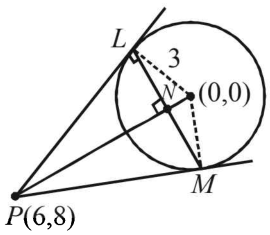
$$\sqrt{(x-a)^2 + (y-b)^2} + \sqrt{x^2 + y^2} = r_1 + r_2,$$

which represents an ellipse whose foci are at (a, b) and $(0, 0)$.

[$\because PS + PS' =$ constant $\Rightarrow$ locus of P is an ellipse with foci at S and S']

22. The given circle is $x^2 + y^2 = r^2$

From pt. (6, 8) tangents are drawn to this circle.



Then length of tangent

$$PL = \sqrt{6^2 + 8^2 - r^2} = \sqrt{100 - r^2}$$

Also equation of chord of contact LM is

$$6x + 8y - r^2 = 0$$

PN = length of $\perp^{lar}$ from P to LM

$$= \frac{36 + 64 - r^2}{\sqrt{36+64}} = \frac{100 - r^2}{10}$$

Now in rt. ΔPLN, $LN^2 = PL^2 - PN^2$

$$= (100 - r^2) - \frac{(100-r^2)^2}{100} = \frac{(100-r^2)r^2}{100}$$

$$\Rightarrow \quad LN = \frac{r\sqrt{100 - r^2}}{10}$$

$$\therefore \quad LM = \frac{r\sqrt{100-r^2}}{5} \qquad (\because LM = 2\,LN)$$

$$\therefore \quad \text{Area of } \Delta PLM = \frac{1}{2} \times LM \times PN$$

$$= \frac{1}{2} \times \frac{r\sqrt{100-r^2}}{5} \times \frac{100-r^2}{10} = \frac{1}{100}\left[r(100-r^2)^{\frac{3}{2}}\right]$$

For max value of area, we should have

$$\frac{dA}{dr} = 0$$

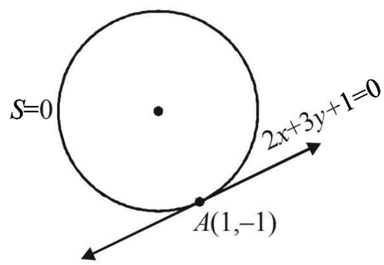
$$\Rightarrow \frac{1}{100}\left[(100-r^2)^{\frac{3}{2}} + r.\frac{3}{2}(100-r^2)^{\frac{1}{2}}(-2r)\right] = 0$$

$$\Rightarrow \quad (100-r^2)^{\frac{1}{2}}[100 - r^2 - 3r^2] = 0 \Rightarrow r = 10 \text{ or } r = 5$$

But $r = 10$ gives length of tangent $PL = 0$

$\therefore \quad r \neq 10$. Hence, $r = 5$

23. We are given that line $2x + 3y + 1 = 0$ touches a circle $S = 0$ at $(1, -1)$.



So, eq^n of this circle can be given by

$$(x-1)^2 + (y+1)^2 + \lambda(2x + 3y + 1) = 0.$$

[**Note :** $(x-1)^2 + (y+1)^2 = 0$ represents a pt. circle with centre at $(1, -1)$].

or $x^2 + y^2 + 2x(\lambda - 1) + y(3\lambda + 2) + (\lambda + 2) = 0$ ...(1)

But given that this circle is orthogonal to the circle, the extremities of whose diameter are $(0, 3)$ and $(-2, -1)$ i.e.

$$x(x+2) + (y-3)(y+1) = 0$$

$$x^2 + y^2 + 2x - 2y - 3 = 0 \qquad \text{..........(2)}$$

Applying the condition of orthogonality for (1) and (2), we

get $2(\lambda - 1).1 + 2\left(\frac{3\lambda + 2}{2}\right).(-1) = \lambda + 2 + (-3)$

$$[2g_1g_2 + 2f_1f_2 = c_1 + c_2]$$

$$\Rightarrow \quad 2\lambda - 2 - 3\lambda - 2 = \lambda - 1$$

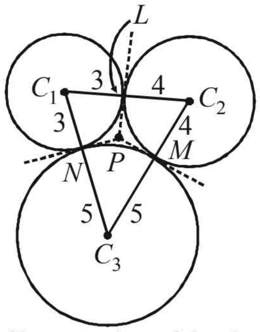
$\Rightarrow\ 2\lambda = -3\ \Rightarrow\ \lambda = \frac{-3}{2}$

Substituting this value of λ in eq^n (1) we get the required circle as

$$x^2 + y^2 - 5x - \frac{5}{2}y + \frac{1}{2} = 0$$

or, $\quad 2x^2 + 2y^2 - 10x - 5y + 1 = 0$

24. Given these circles with centres at C_1, C_2 and C_3 and with radii 3, 4 and 5 respectively, The three circles touch each other externally as shown in the figure.



P is the point of intersection of the three tangents drawn at the pts of contacts, L, M and N. Since lengths of tangents to a circle from a point are equal, we get

$PL = PM = PN$

Also $PL \perp C_1C_2, PM \perp C_2C_3, PN \perp C_1C_3$

($\because$ tangent is perpendicular to the radius at pt. of contact)

Clearly P is the incentre of $\Delta C_1C_2C_3$ and its distance from pt.of contact i.e., PL is the radius of incircle of $\Delta C_1C_2C_3$.

In $\Delta C_1C_2C_3$ sides are

$a = 3 + 4 = 7, b = 4 + 5 = 9, c = 5 + 3 = 8$

$\therefore \quad s = \frac{a+b+c}{2} = 12$

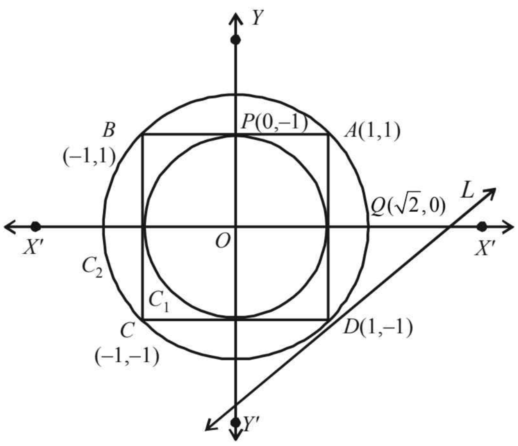
$\therefore \quad \Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{12 \times 5 \times 3 \times 4} = 12\sqrt{5}$

$\therefore \quad r = \frac{\Delta}{s} = \frac{12\sqrt{5}}{12} = \sqrt{5}$

G. Comprehension Based Questions

1. (a) Without loss of generality we can assume the square $ABCD$ with its vertices A (1, 1), B (–1, 1), C(–1, –1), D (1,–1)

P to be the point (0, 1) and Q as $(\sqrt{2}, 0)$.



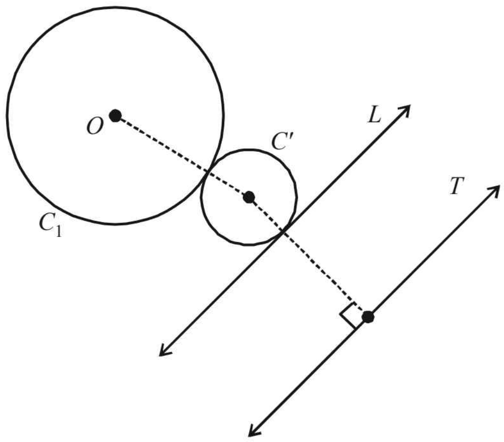
Then, $\frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$

$$= \frac{1+1+5+5}{2[(\sqrt{2}-1)^2+1]+2((\sqrt{2}+1)^2+1]} = \frac{12}{16} = 0.75$$

2. (b) Let C' be the said circle touching C_1 and L, so that C_1 and C' are on the same side of L. Let us draw a line T parallel to L at a distance equal to the radius of circle C_1, on opposite side of L.

Then the centre of C' is equidistant from the centre of C_1 and from line T.

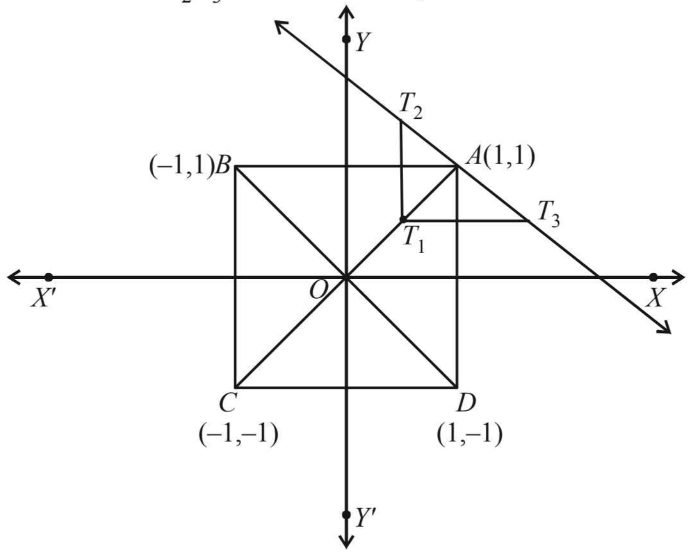
$\Rightarrow$ locus of centre of C' is a parabola.



3. (c) Since S is equidistant form A and line BD, it traces a parabola. Clearly, AC is the axis, A (1, 1) is the focus and $T_1\left(\frac{1}{2},\frac{1}{2}\right)$ is the vertex of parabola.

$AT_1 = \frac{1}{\sqrt{2}}$.

T_2T_3 = latus rectum of parabola



$= 4 \times \frac{1}{\sqrt{2}} = 2\sqrt{2}$

$\therefore \quad$ Area $(\Delta T_1T_2T_3) = \frac{1}{2} \times \frac{1}{\sqrt{2}} \times 2\sqrt{2} = \frac{1}{2} = 1$ sq. units

4. **(d)** Slope of $CD = \frac{1}{\sqrt{3}}$

$\therefore$ Parametric equation of CD is

$$\frac{x-\frac{3\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} = \frac{y-\frac{3}{2}}{\frac{1}{2}} = \pm 1$$

$\therefore$ Two possible coordinates of C are

$$\left(\frac{\sqrt{3}}{2}+\frac{3\sqrt{3}}{2}, \frac{1}{2}+\frac{3}{2}\right) \text{ or } \left(\frac{-\sqrt{3}}{2}+\frac{3\sqrt{3}}{2}, -\frac{1}{2}+\frac{3}{2}\right)$$

i.e. $(2\sqrt{3}, 2)$ or $(\sqrt{3}, 1)$

As (0, 0) and C lie on the same side of PQ

$\therefore$ $(\sqrt{3}, 1)$ should be the coordinates of C.

NOTE THIS STEP : Remember (x_1, y_1) and (x_2, y_2) lie on the same or opposite side of a line ax + by + c = 0

according as $\frac{ax_1+by_1+c}{ax_2+by_2+c} > 0$ or < 0.

$\therefore$ Equation of the circle is

$$(x-\sqrt{3})^2+(y-1)^2 = 1$$

5. **(a)** ΔPQR is an equilateral triangle, the incentre C must coincide with centroid of ΔPQR and D, E, F must concide with the mid points of sides PQ, QR and RP respectively.

Also $\angle CPD = 30° \Rightarrow PD = \sqrt{3}$

Writing the equation of side PQ in symmetric form we get,

$$\frac{x-\frac{3\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{y-\frac{3}{2}}{\frac{\sqrt{3}}{2}} = \mp\sqrt{3}$$

$\therefore$ Coordinates of $P = \left(\frac{\sqrt{3}}{2}+\frac{3\sqrt{3}}{2}, \frac{-3}{2}+\frac{3}{2}\right)$

$= (2\sqrt{3}, 0)$ and

coordinates of $Q = \left(\frac{-\sqrt{3}}{2}+\frac{3\sqrt{3}}{2}, \frac{3}{2}+\frac{3}{2}\right) = (\sqrt{3}, 3)$

Let coordinates of R be (α, β), then using the formula for centroid of Δ we get

$$\frac{\sqrt{3}+2\sqrt{3}+\alpha}{3} = \sqrt{3} \text{ and } \frac{3+0+\beta}{3} = 1$$

$\Rightarrow \alpha = 0$ and $\beta = 0$

$\therefore$ Coordinates of R = (0, 0)

Now coordinates of E = mid point of QR = $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$

and coordinates of F = mid point of PR = $(\sqrt{3}, 0)$

6. **(d)** Equation of side QR is $y = \sqrt{3}x$ and equation of side RP is $y = 0$

Paragraph 3

Given the implicit function $y^3 - 3y + x = 0$

For $x \in (-\infty, -2) \cup (2, \infty)$ it is y = f (x) real valued differentiable function and for $x \in (-2, 2)$ it is y = g(x) real valued differentiable function.

7. **(a)** Equation of tangent PT to the circle $x^2+y^2=4$ at the point $P(\sqrt{3}, 1)$ is $x\sqrt{3}+y=4$

Let the line L, perpendicular to tangent PT be

$$x - y\sqrt{3} + \lambda = 0$$

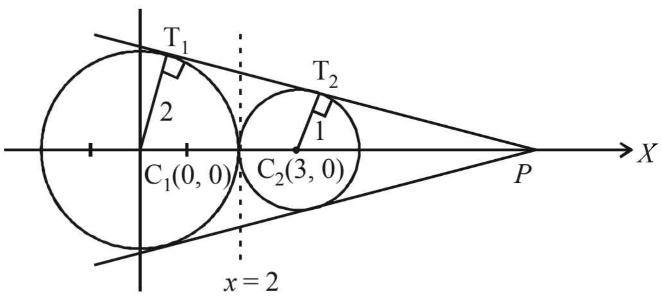
As it is tangent to the circle $(x-3)^2+y^2=1$

$\therefore$ length of perpendicular from centre of circle to the tangent = radius of circle.

$$\Rightarrow \left|\frac{3+\lambda}{2}\right| = 1 \Rightarrow \lambda = -1 \text{ or } -5$$

$\therefore$ Equation of L can be $x-\sqrt{3}y = 1$ or $x-\sqrt{3}y = 5$

8. **(d)**



From the figure it is clear that the intersection point of two direct common tangents lies on x-axis.

Also $\Delta PT_1C_1 \sim \Delta PT_2C_2$

$\Rightarrow PC_1 : PC_2 = 2 : 1$

or P divides C_1C_2 in the ratio 2 : 1 externally

$\therefore$ Coordinates of P are (6, 0)

Let the equation of tangent through P be

$y = m(x-6)$

As it touches $x^2+y^2=4$

$$\therefore \left|\frac{6m}{\sqrt{m^2+1}}\right| = 2 \Rightarrow 36m^2 = 4(m^2+1)$$

$$\Rightarrow m = \pm\frac{1}{2\sqrt{2}}$$

$\therefore$ Equations of common tangents are

$$y = \pm\frac{1}{2\sqrt{2}}(x-6)$$

Also $x = 2$ is the common tangent to the two circles.

H. Assertion & Reason Type Questions

1. **(a)** Equation of director circle of the given circle $x^2+y^2=169$ is $x^2+y^2 = 2\times169 = 338$.

We know from every point on director circle, the tangents drawn to given circle are perpendicular to each other.

Here (17, 7) lies on director circle.

$\therefore$ The tangent from (17, 7) to given circle are mutually perpendicular.

2. (c) The given circle is $x^2+y^2+6x-10y+30=0$

Centre (–3, 5), radius = 2

$L_1: 2x+3y+(p-3)=0$;

$L_2: 2x+3y+p+3=0$

Clearly $L_1 \| L_2$

Distance between L_1 and L_2

$$=\left|\frac{p+3-p+3}{\sqrt{2^2+3^2}}\right|=\frac{6}{\sqrt{13}}<2$$

$\Rightarrow$ If one line is a chord of the given circle, other line may or may not the diameter of the circle.

$\therefore$ Statement 1 is true and statement 2 is false.

I. Integer Value Correct Type

1. (8) Let r be the radius of required circle.

Clearly, in ΔC_1CC_2, $C_1C=C_2C=r+1$

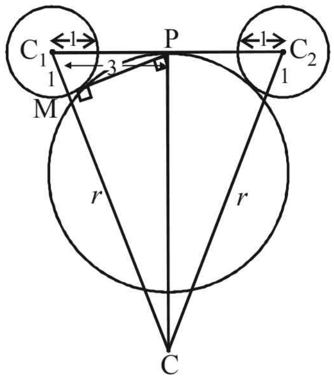
and P is mid point of C_1C_2

$\therefore$ $CP \perp C_1C_2$

Also $PM \perp CC_1$

Now $\Delta PMC_1 \sim \Delta CPC_1$ (by AA similarity)

$$\therefore \frac{MC_1}{PC_1}=\frac{PC_1}{CC_1}$$



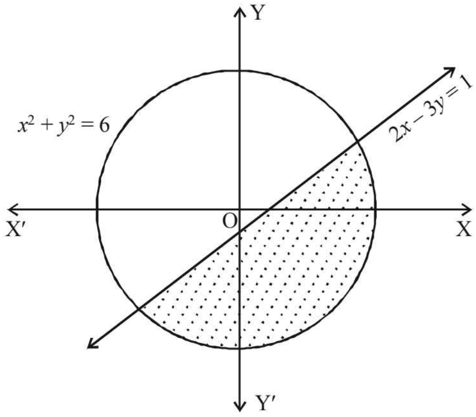
$$\Rightarrow \frac{1}{3}=\frac{3}{r+1} \Rightarrow r+1=9 \Rightarrow r=8.$$

2. (2)

The smaller region of circle is the region given by

$x^2+y^2 \le 6$ ...(1)

and $2x-3y \ge 1$ ...(2)



We observe that only two points $\left(2,\frac{3}{4}\right)$ and $\left(\frac{1}{4},-\frac{1}{4}\right)$ satisfy both the inequations (1) and (2)

$\therefore$ 2 points in S lie inside the smaller part.

Section-B JEE Main/ AIEEE

1. (c) Equation of circle $x^2+y^2=1=(1)^2$

$\Rightarrow x^2+y^2=(y-mx)^2 \Rightarrow x^2=m^2x^2-2\,mxy$;

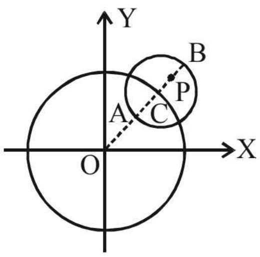
$\Rightarrow x^2(1-m^2)+2mxy=0$. Which represents the pair of lines between which the angle is 45°.

$$\tan 45=\pm\frac{2\sqrt{m^2-0}}{1-m^2}=\frac{\pm 2m}{1-m^2};$$

$$\Rightarrow 1-m^2=\pm 2m \Rightarrow m^2\pm 2m-1=0$$

$$\Rightarrow m=\frac{-2\pm\sqrt{4+4}}{2}=\frac{-2\pm 2\sqrt{2}}{2}=-1\pm\sqrt{2}.$$

2. (a) For any point P (x, y) in the given circle,



we should have

$$OA \le OP \le OB \Rightarrow (5-3) \le \sqrt{x^2+y^2} \le 5+3$$

$$\Rightarrow 4 \le x^2+y^2 \le 64$$

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3. **(b)** Let the required circle be
$x^2 + y^2 + 2gx + 2fy + c = 0$
Since it passes through (0, 0) and (1, 0)

$\Rightarrow c = 0$ and $g = -\frac{1}{2}$

Points (0, 0) and (1, 0) lie inside the circle $x^2 + y^2 = 9$, so two circles touch internally
$\Rightarrow c_1c_2 = r_1 - r_2$

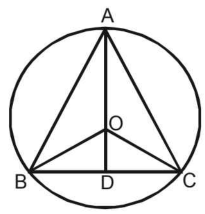
$\therefore \sqrt{g^2+f^2} = 3 - \sqrt{g^2+f^2} \Rightarrow \sqrt{g^2+f^2} = \frac{3}{2}$

$\Rightarrow f^2 = \frac{9}{4} - \frac{1}{4} = 2 \qquad \therefore f = \pm\sqrt{2}$.

Hence, the centres of required circle are

$\left(\frac{1}{2}, \sqrt{2}\right)$ or $\left(\frac{1}{2}, -\sqrt{2}\right)$

4. **(c)** Let ABC be an equilateral triangle, whose median is AD.



Given $AD = 3a$.
In ΔABD, $AB^2 = AD^2 + BD^2$;
$\Rightarrow x^2 = 9a^2 + (x^2/4)$ where $AB = BC = AC = x$.

$\frac{3}{4}x^2 = 9a^2 \Rightarrow x^2 = 12a^2$.

In ΔOBD, $OB^2 = OD^2 + BD^2$

$\Rightarrow r^2 = (3a - r)^2 + \frac{x^2}{4}$

$\Rightarrow r^2 = 9a^2 - 6ar + r^2 + 3a^2$; $\Rightarrow 6ar = 12a^2$
$\Rightarrow r = 2a$
So equation of circle is $x^2 + y^2 = 4a^2$

5. **(b)** $|r_1 - r_2| < C_1C_2$ for intersection

$\Rightarrow r - 3 < 5 \Rightarrow r < 8 \qquad$ ...(1)

and $r_1 + r_2 > C_1C_2$, $r + 3 > 5 \Rightarrow r > 2 \quad$ ...(2)

From (1) and (2), $2 < r < 8$.

6. **(d)** $\pi r^2 = 154 \Rightarrow r = 7$
For centre on solving equation
$2x - 3y = 5$ & $3x - 4y = 7$ we get $x = 1, y = -1$
$\therefore$ centre = (1, –1)

Equation of circle, $(x-1)^2 + (y+1)^2 = 7^2$

$x^2 + y^2 - 2x + 2y = 47$

7. **(b)** Let the variable circle is

$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad$ ......(1)

It passes through (a, b)

$\therefore a^2 + b^2 + 2ga + 2fb + c = 0 \qquad$ ......(2)

(1) cuts $x^2 + y^2 = 4$ orthogonally

$\therefore 2(g \times 0 + f \times 0) = c - 4 \Rightarrow c = 4$

$\therefore$ from (2) $a^2 + b^2 + 2ga + 2fb + 4 = 0$

$\therefore$ Locus of centre $(-g, -f)$ is

$a^2 + b^2 - 2ax - 2by + 4 = 0$

or $2ax + 2by = a^2 + b^2 + 4$

8. **(d)** Let the variable circle be

$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad$ ....(1)

$\therefore p^2 + q^2 + 2gp + 2fq + c = 0 \qquad$ ....(2)

Circle (1) touches x-axis,

$\therefore g^2 - c = 0 \Rightarrow c = g^2$. From (2)

$p^2 + q^2 + 2gp + 2fq + g^2 = 0 \qquad$ ....(3)

Let the other end of diameter through (p, q) be (h, k),

then, $\frac{h+p}{2} = -g$ and $\frac{k+q}{2} = -f$

Put in (3)

$$p^2 + q^2 + 2p\left(-\frac{h+p}{2}\right) + 2q\left(-\frac{k+q}{2}\right) + \left(\frac{h+p}{2}\right)^2 = 0$$

$\Rightarrow h^2 + p^2 - 2hp - 4kq = 0$

$\therefore$ locus of (h, k) is $x^2 + p^2 - 2xp - 4yq = 0$

$\Rightarrow (x - p)^2 = 4qy$

9. **(d)** Two diameters are along

$2x + 3y + 1 = 0$ and $3x - y - 4 = 0$

solving we get centre (1, –1)
circumference = $2\pi r = 10\pi$
$\therefore r = 5$.

Required circle is, $(x-1)^2 + (y+1)^2 = 5^2$

$\Rightarrow x^2 + y^2 - 2x + 2y - 23 = 0$

10. **(d)** Solving $y = x$ and the circle

$x^2 + y^2 - 2x = 0$, we get

$x = 0, y = 0$ and $x = 1, y = 1$

$\therefore$ Extremities of diameter of the required circle are (0, 0) and (1, 1). Hence, the equation of circle is

$(x-0)(x-1) + (y-0)(y-1) = 0$

$\Rightarrow x^2 + y^2 - x - y = 0$

11. **(b)** $s_1 = x^2 + y^2 + 2ax + cy + a = 0$

$s_2 = x^2 + y^2 - 3ax + dy - 1 = 0$

Equation of common chord of circles s_1 and s_2 is

given by $s_1 - s_2 = 0$

$\Rightarrow 5ax + (c - d)y + a + 1 = 0$

Given that $5x + by - a = 0$ passes through P and Q

$\therefore$ The two equations should represent the same line

$\Rightarrow \frac{a}{1} = \frac{c-d}{b} = \frac{a+1}{-a} \Rightarrow a+1 = -a^2$

$a^2 + a + 1 = 0$

No real value of a.

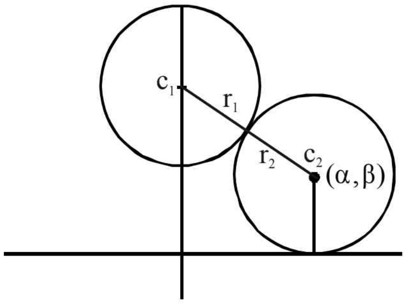
12. (d) Equation of circle with centre (0, 3) and radius 2 is

$x^2 + (y-3)^2 = 4$

Let locus of the variable circle is (α, β)

$\because$ It touches x - axis.

$\therefore$ It's equation is $(x-\alpha)^2 + (y+\beta)^2 = \beta^2$



Circle touch externally $\Rightarrow c_1c_2 = r_1 + r_2$

$\therefore \sqrt{\alpha^2 + (\beta-3)^2} = 2 + \beta$

$\alpha^2 + (\beta-3)^2 = \beta^2 + 4 + 4\beta \Rightarrow \alpha^2 = 10(\beta - 1/2)$

$\therefore$ Locus is $x^2 = 10\left(y - \frac{1}{2}\right)$ which is a parabola.

13. (d) Let the centre be (α, β)

$\because$ It cuts the circle $x^2 + y^2 = p^2$ orthogonally

$\therefore$ Using $2g_1g_2 + 2f_1f_2 = c_1 + c_2$, we get

$2(-\alpha) \times 0 + 2(-\beta) \times 0 = c_1 - p^2 \Rightarrow c_1 = p^2$

Let equation of circle is

$x^2 + y^2 - 2\alpha x - 2\beta y + p^2 = 0$

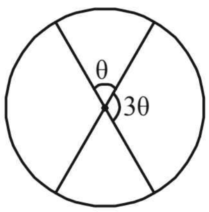
It passes through

$(a, b) \Rightarrow a^2 + b^2 - 2\alpha a - 2\beta b + p^2 = 0$

$\therefore$ Locus of (α, β) is

$\therefore 2ax + 2by - (a^2 + b^2 + p^2) = 0$.

14. (d)



As per question area of one sector = 3 area of another sector

$\Rightarrow$ angle at centre by one sector $= 3 \times$ angle at centre by another sector

Let one angle be θ then other $= 3\theta$

Clearly $\theta + 3\theta = 180 \Rightarrow \theta = 45°$

$\therefore$ Angle between the diameters represented by combined equation

$ax^2 + 2(a+b)xy + by^2 = 0$ is $45°$

$\therefore$ Using $\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$

we get $\tan 45° = \frac{2\sqrt{(a+b)^2 - ab}}{a+b}$

$\Rightarrow 1 = \frac{2\sqrt{a^2 + b^2 + ab}}{a+b} \Rightarrow (a+b)^2 = 4(a^2 + b^2 + ab)$

$\Rightarrow a^2 + b^2 + 2ab = 4a^2 + 4b^2 + 4ab$

$\Rightarrow 3a^2 + 3b^2 + 2ab = 0$

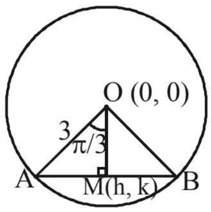
15. (d) Point of intersection of $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ is $(1, -1)$ which is the centre of the circle and radius = 7

$\therefore$ Equation is $(x-1)^2 + (y+1)^2 = 49$

$\Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$

16. (d) Let $M(h, k)$ be the mid point of chord AB where

$\angle AOB = \frac{2\pi}{3}$



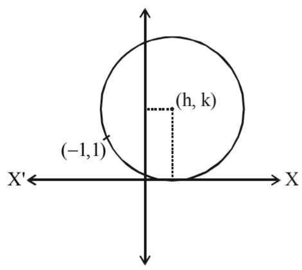
$\therefore \angle AOM = \frac{\pi}{3}$. Also $OM = 3\cos\frac{\pi}{3} = \frac{3}{2}$

$\Rightarrow \sqrt{h^2 + k^2} = \frac{3}{2} \Rightarrow h^2 + k^2 = \frac{9}{4}$

$\therefore$ Locus of (h, k) is $x^2 + y^2 = \frac{9}{4}$

17. (d) Equation of circle whose centre is (h, k)

i.e $(x-h)^2 + (y-k)^2 = k^2$



(radius of circle = k because circle is tangent to x-axis)

Equation of circle passing through $(-1, +1)$

$\therefore (-1-h)^2 + (1-k)^2 = k^2$

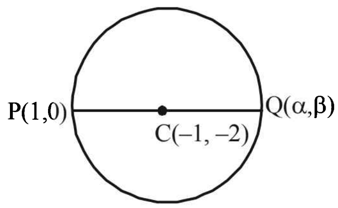
$\Rightarrow 1 + h^2 + 2h + 1 + k^2 - 2k = k^2 \Rightarrow h^2 + 2h - 2k + 2 = 0$

$D \geq 0$

$\therefore (2)^2 - 4 \times 1.(-2k + 2) \geq 0$

$\Rightarrow 4 - 4(-2k + 2) \geq 0 \Rightarrow 1 + 2k - 2 \geq 0 \Rightarrow k \geq \frac{1}{2}$

18. (c) The given circle is $x^2 + y^2 + 2x + 4y - 3 = 0$



Centre $(-1, -2)$

Let $Q(\alpha, \beta)$ be the point diametrically opposite to the point $P(1, 0)$,

then $\frac{1+\alpha}{2} = -1$ and $\frac{0+\beta}{2} = -2$

$\Rightarrow \alpha = -3, \beta = -4$, So, Q is $(-3, -4)$

19. (c) Let the centre of the circle be $(h, 2)$

$\therefore$ Equation of circle is

$(x-h)^2 + (y-2)^2 = 25$ ...(1)

Differentiating with respect to x, we get

$$2(x-h) + 2(y-2)\frac{dy}{dx} = 0$$

$$\Rightarrow x - h = -(y-2)\frac{dy}{dx}$$

Substituting in equation (1) we get

$$(y-2)^2\left(\frac{dy}{dx}\right)^2 + (y-2)^2 = 25$$

$\Rightarrow (y-2)^2 (y')^2 = 25 - (y-2)^2$

20. (a) The given circles are

$S_1 \equiv x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ ....(1)

$S_2 \equiv x^2 + y^2 + 2x + 2y - p^2 = 0$ ....(2)

$\therefore$ Equation of common chord PQ is $S_1 - S_2 = 0$

$\Rightarrow$ $L \equiv x + 5y + p^2 + 2p - 5 = 0$

$\Rightarrow$ Equation of circle passing through P and Q is

$S_1 + \lambda L = 0$

$\Rightarrow (x^2 + y^2 + 3x + 7y + 2p - 5) + \lambda (x + 5y + p^2 + 2p - 5) = 0$

As it passes through (1, 1), therefore

$\Rightarrow$ $(7 + 2p) + \lambda (2p + p^2 + 1) = 0$

$\Rightarrow \lambda = -\frac{2p+7}{(p+1)^2}$, which does not exist for $p = -1$

21. (a) Circle $x^2 + y^2 - 4x - 8y - 5 = 0$

Centre = (2, 4), Radius = $\sqrt{4+16+5} = 5$

If circle is intersecting line $3x - 4y = m$, at two distinct points.

$\Rightarrow$ length of perpendicular from centre to the line < radius

$$\Rightarrow \frac{|6-16-m|}{5} < 5 \Rightarrow |10+m| < 25$$

$\Rightarrow -25 < m + 10 < 25 \Rightarrow -35 < m < 15$

22. (a) As centre of one circle is (0, 0) and other circle passes through (0, 0), therefore

Also $C_1\left(\frac{a}{2}, 0\right)$ $C_2(0, 0)$

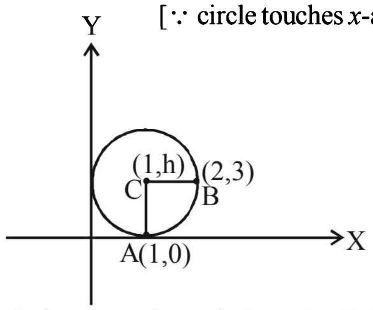
$r_1 = \frac{a}{2}$ $r_2 = C$

$C_1C_2 = r_1 - r_2 = \frac{a}{2} \Rightarrow C - \frac{a}{2} = \frac{a}{2} \Rightarrow C = a$

If the two circles touch each other, then they must touch each other internally.

23. (a) Let centre of the circle be $(1, h)$

[$\because$ circle touches x-axis at (1,0)]



Let the circle passes through the point B (2,3)

$\therefore$ $CA = CB$ (radius)

$\Rightarrow$ $CA^2 = CB^2$

$\Rightarrow$ $(1-1)^2 + (h-0)^2 = (1-2)^2 + (h-3)^2$

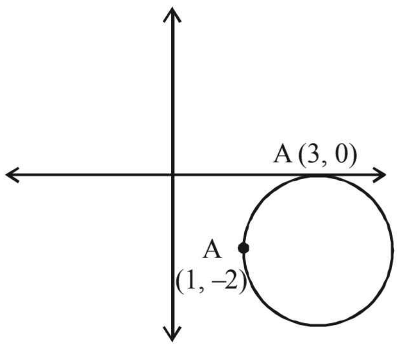
$\Rightarrow$ $h^2 = 1 + h^2 + 9 - 6h \Rightarrow h = \frac{10}{6} = \frac{5}{3}$

Thus, diameter is $2h = \frac{10}{3}$.

24. (c) Since circle touches x-axis at (3, 0)

$\therefore$ The equation of circle be

$(x-3)^2 + (y-0)^2 + \lambda y = 0$



As it passes through (1, –2)

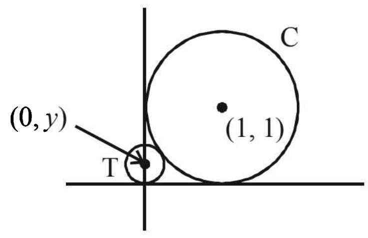
$\therefore$ Put $x = 1, y = -2$

$\Rightarrow$ $(1-3)^2 + (-2)^2 + \lambda(-2) = 0 \Rightarrow \lambda = 4$

$\therefore$ equation of circle is $(x-3)^2 + y^2 - 8 = 0$

Now, from the options (5, –2) satisfies equation of circle.

25. (b)



Equation of circle $C \equiv (x-1)^2 + (y-1)^2 = 1$

Radius of T = $|y|$
T touches *C* externally
therefore,
Distance between the centres = sum of their radii

$\Rightarrow \sqrt{(0-1)^2+(y-1)^2} = 1+|y|$

$\Rightarrow (0-1)^2+(y-1)^2 = (1+|y|)^2$

$\Rightarrow 1+y^2+1-2y = 1+y^2+2|y|$

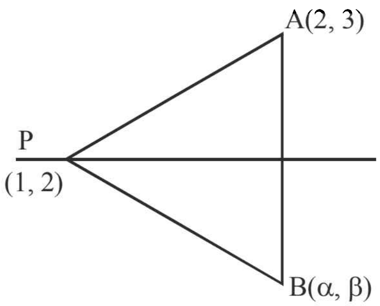
$2|y| = 1-2y$

If $y > 0$ then $2y = 1-2y \Rightarrow y = \frac{1}{4}$

If $y < 0$ then $-2y = 1-2y \Rightarrow 0 = 1$
(not possible)

$\therefore \quad y = \frac{1}{4}$

26. (a) Intersection point of $2x-3y+4=0$ and $x-2y+3=0$ is $(1, 2)$



Since, P is the fixed point for given family of lines

So, PB = PA

$(\alpha-1)^2+(\beta-2)^2 = (2-1)^2+(3-2)^2$

$(\alpha-1)^2+(\beta-2)^2 = 1+1 = 2$

$(x-1)^2+(y-2)^2 = (\sqrt{2})^2$

$(x-a)^2+(y-b)^2 = r^2$

Therefore, given locus is a circle with centre (1, 2) and radius $\sqrt{2}$.

27. (a) $x^2+y^2-4x-6y-12=0$ ...(i)

Centre, $c_1 = (2, 3)$ and Radius, $r_1 = 5$ units

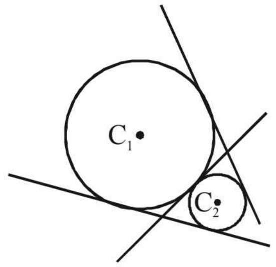
$x^2+y^2+6x+18y+26=0$ ...(ii)

Centre, $c_2 = (-3, -9)$ and Radius, $r_2 = 8$ units

$C_1C_2 = \sqrt{(2+3)^2+(3+9)^2} = 13$ units

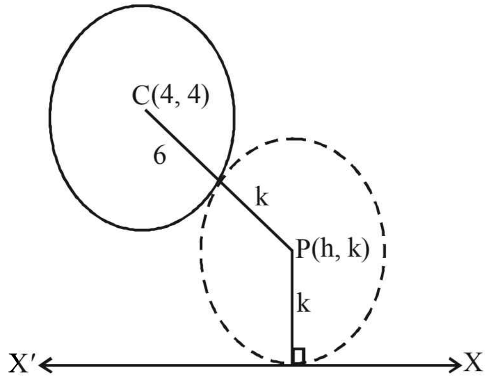
$r_1+r_2 = 5+8 = 13$

$\therefore C_1C_2 = r_1+r_2$



Therefore there are three common tangents.

28. (b)



For the given circle,
centre : (4, 4)
radius = 6

$6+k = \sqrt{(h-4)^2+(k-4)^2}$

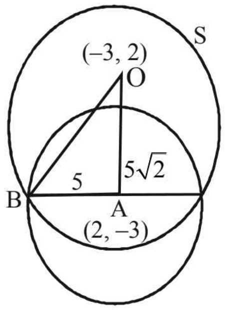
$(h-4)^2 = 20k+20$

$\therefore$ locus of (h, k) is

$(x-4)^2 = 20(y+1)$,

which is a parabola.

29. (d)



Centre of S : O (–3, 2) centre of given circle A(2, –3)

$\Rightarrow OA = 5\sqrt{2}$

Also AB = 5 ($\because$ AB = r of the given circle)

$\Rightarrow$ Using pythagoras theorem in ΔOAB

$r = 5\sqrt{3}$